

Rational Expression Review

1.

$$\begin{aligned} \text{a) } \frac{5}{6} + \frac{1}{3} & \quad \frac{1}{3} \text{ (2)} \\ & = \frac{5}{6} + \frac{2}{6} \\ & = \frac{7}{6} \end{aligned}$$

$$\begin{aligned} \text{b) } \frac{a}{b} + \frac{c}{ab} & \quad \frac{a}{b} \text{ (a)} \\ & = \frac{a^2}{ab} + \frac{c}{ab} \\ & = \frac{a^2 + c}{ab} \end{aligned}$$

$$\begin{aligned} \text{c) } \frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} & \quad \frac{\sin \theta}{\cos \theta} \text{ (sin } \theta) \\ & = \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} \end{aligned}$$

$$\begin{aligned} \text{d) } \frac{1}{\cos^2 \theta} + \frac{1}{\cos \theta} & \quad \frac{1}{\cos \theta} \text{ (cos } \theta) \\ & = \frac{1 + \cos \theta}{\cos^2 \theta} \end{aligned}$$

$$\begin{aligned} \text{e) } \frac{2}{x} + \frac{3}{y} & \quad \frac{2}{x} \text{ (y)} \\ & = \frac{2y + 3x}{xy} \\ & = \frac{3x + 2y}{xy} \end{aligned}$$

$$\begin{aligned} \text{f) } \frac{7}{xy^2} - \frac{15}{x^2y} & \quad \frac{7}{xy^2} \text{ (x)} \\ & = \frac{7x - 15y}{x^2y^2} \end{aligned}$$

$$\begin{aligned} \text{g) } \frac{\cos \theta}{\sin \theta} - \frac{1}{\sin^2 \theta} & \quad \frac{\cos \theta}{\sin \theta} \text{ (sin } \theta) \\ & = \frac{\sin \theta \cos \theta - 1}{\sin^2 \theta} \end{aligned}$$

$$\begin{aligned} \text{h) } \frac{1}{\sin \theta \cos \theta} - \frac{1}{\cos \theta \sin \theta} & \quad \frac{1}{\cos \theta \sin \theta} \text{ (same!)} \\ & = \frac{1}{\sin \theta \cos \theta} \end{aligned}$$

$$\begin{aligned} \text{i) } \frac{17a}{bc} + \frac{5b}{ac} & \quad \frac{17a}{bc} \text{ (a)} \\ & = \frac{17a^2 + 5b^2}{abc} \end{aligned}$$

$$\begin{aligned} \text{j) } \frac{15n^2}{m} + \frac{11n}{m^2} & \quad \frac{15n^2}{m} \text{ (m)} \\ & = \frac{15mn^2 + 11n}{m^2} \\ & = \frac{n(15mn + 11)}{m^2} \end{aligned}$$

$$\begin{aligned} \text{k) } \frac{\sin \theta}{\cos^2 \theta} - \frac{\cos \theta}{\sin \theta} & \quad \frac{\sin \theta}{\cos^2 \theta} \text{ (sin } \theta) \\ & = \frac{\sin^2 \theta - \cos^3 \theta}{\sin \theta \cos^2 \theta} \end{aligned}$$

$$\begin{aligned} \text{l) } \frac{2\sin \theta}{\cos^2 \theta} - \frac{\sin \theta}{\cos \theta} & \quad \frac{\sin \theta}{\cos \theta} \text{ (cos } \theta) \\ & = \frac{2\sin \theta - \sin \theta \cos \theta}{\cos^2 \theta} \\ & = \frac{\sin \theta (2 - \cos \theta)}{\cos^2 \theta} \end{aligned}$$

$$\begin{aligned} \text{2.l) } \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta} & \quad \frac{1}{\cos \theta} \text{ (sin } \theta \cos \theta) \\ & = \frac{\frac{1}{\cos \theta} + \frac{\sin \theta}{\sin \theta}}{\sin \theta} \\ & = \frac{\frac{\sin \theta \cos \theta + \sin \theta \cos^2 \theta}{\cos \theta}}{\sin \theta} \\ & = \frac{\sin^2 \theta + \sin \theta}{\sin \theta + \cos^2 \theta} = \frac{\sin \theta (\sin \theta + 1)}{\sin \theta + \cos^2 \theta} \end{aligned}$$

2.

$$\begin{aligned} a) & \frac{\left(\frac{2}{3} + 1\right)^6}{\left(\frac{1}{2} + \frac{1}{3}\right)^6} \\ & = \frac{\frac{12}{3} + 6}{\frac{6}{2} + \frac{6}{3}} \\ & = \frac{4 + 6}{3 + 2} \\ & = \boxed{\frac{10}{5}} = 2 \end{aligned}$$

$$\begin{aligned} b) & \frac{\left(\frac{x}{y} + x\right)^{xy}}{\left(\frac{y}{x} + y\right)^{xy}} \\ & = \frac{x^2 y + x^2 y}{\frac{xy^2}{x} + xy^2} \\ & = \frac{x^2 + x^2 y}{y^2 + xy^2} = \frac{x^2(1+y)}{y^2(1+x)} \end{aligned}$$

$$\begin{aligned} c) & \frac{\frac{\cos\theta}{\sin\theta} + \sin\theta}{\left(\frac{1}{\sin\theta} + 1\right)\sin\theta} \\ & = \frac{\cos\theta\sin\theta + \sin^2\theta}{\frac{\sin\theta}{\sin\theta} + \sin\theta} \\ & = \frac{\cos\theta + \sin^2\theta}{1 + \sin\theta} \end{aligned}$$

$$\begin{aligned} d) & \frac{\left(\cos\theta + \frac{1}{\sin\theta}\right)\sin\theta}{(\sin\theta)\sin\theta} \\ & = \frac{\cos\theta\sin\theta + \frac{\sin\theta}{\sin\theta}}{\sin^2\theta} \\ & = \boxed{\frac{\cos\theta\sin\theta + 1}{\sin^2\theta}} \end{aligned}$$

$$\begin{aligned} e) & \frac{\left(x + \frac{2x}{3}\right)^3}{(6)^3} \\ & = \frac{3x + \frac{6x}{3}}{18} \\ & = \frac{3x + 2x}{18} \\ & = \boxed{\frac{5x}{18}} \end{aligned}$$

$$\begin{aligned} f) & \frac{\left(6x - \frac{3x}{5}\right)^5}{(3x)^5} \\ & = \frac{30x - \frac{15x}{5}}{15x} \\ & = \frac{30x - 3x}{15x} \\ & = \frac{27x}{15x} = \boxed{\frac{9}{5}} \end{aligned}$$

$$\begin{aligned} g) & \frac{\left(5s - \frac{3s}{4}\right)^4}{\left(s - \frac{5s}{4}\right)^4} \\ & = \frac{20s - \frac{12s}{4}}{4s - \frac{20s}{4}} \\ & = \frac{20s - 3s}{4s - 5s} \\ & = \frac{17s}{-s} = \frac{17}{-1} = \boxed{-17} \end{aligned}$$

$$\begin{aligned} h) & \frac{\left(\frac{\cos\theta}{\sin\theta} - 1\right)\sin\theta}{\left(1 - \frac{\cos\theta}{\sin\theta}\right)\sin\theta} \\ & = \frac{\frac{\cos\theta\sin\theta}{\sin\theta} - \sin\theta}{\sin\theta - \frac{\cos\theta\sin\theta}{\sin\theta}} \\ & = \frac{\cos\theta - \sin\theta}{\sin\theta - \cos\theta} \\ & = \frac{-1(\sin\theta - \cos\theta)}{\sin\theta - \cos\theta} \\ & = \boxed{-1} \end{aligned}$$

$$\begin{aligned} i) & \frac{\left(8a + 5 - \frac{3a}{2}\right)^2}{\left(3a - 7 + \frac{5a}{2}\right)^2} \\ & = \frac{16a + 10 - \frac{6a}{2}}{6a - 14 + \frac{10a}{2}} \\ & = \frac{16a + 10 - 3a}{6a - 14 + 5a} \\ & = \boxed{\frac{13a + 10}{11a - 14}} \end{aligned}$$

$$\begin{aligned} j) & \frac{\left(2m + \frac{5}{m-3}\right)^{m-3}}{\left(5m - \frac{2}{m-3}\right)^{m-3}} \\ & = \frac{2m(m-3) + \frac{5(m-3)}{m-3}}{5m(m-3) - \frac{2(m-3)}{m-3}} \\ & = \boxed{\frac{2m^2 - 6m + 5}{5m^2 - 15m - 2}} \end{aligned}$$

$$\begin{aligned} k) & \frac{\left(2s - 5 - \frac{2s^2 - 3s}{s+1}\right)(s+1)}{\left(3s - 1 - \frac{2s+1}{s+1}\right)(s+1)} \\ & = \frac{(2s-5)(s+1) - (2s^2-3s)(s+1)}{(3s-1)(s+1) - (2s+1)(s+1)} \\ & = \frac{2s^2 + 2s - 5s - 5 - 2s^3 + 3s}{3s^2 + 3s - s - 1 - 2s - 1} \\ & = \boxed{\frac{-5}{3s^2 - 2}} \end{aligned}$$

see over
for #2.1)