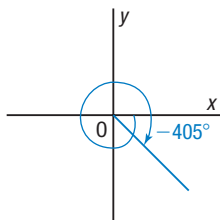


Checkpoint: Assess Your Understanding, pages 561–565

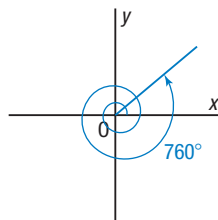
8.1

1. a) Sketch each angle θ in standard position.

i) $\theta = -405^\circ$



ii) $\theta = 760^\circ$



b) For each angle θ in part a

i) Determine the measures of angles that are coterminal with θ in the domain $-800^\circ \leq \theta \leq 800^\circ$.

i) The measures of the angles coterminal with -405° are:
 -405°
 $-405^\circ + 360^\circ = -45^\circ$
 $-405^\circ + 2(360^\circ) = 315^\circ$
 $-405^\circ + 3(360^\circ) = 675^\circ$
 and $-405^\circ - 360^\circ = -765^\circ$
 The measures of all angles coterminal with -405° are:
 $-405^\circ + k360^\circ, k \in \mathbb{Z}$

ii) Write an expression for all angles that are coterminal with θ .

ii) The measures of the angles coterminal with 760° are:
 760°
 $760^\circ - 360^\circ = 400^\circ$
 $760^\circ - 2(360^\circ) = 40^\circ$
 $760^\circ - 3(360^\circ) = -320^\circ$
 and $760^\circ - 4(360^\circ) = -680^\circ$
 The measures of all angles coterminal with 760° are:
 $760^\circ + k360^\circ, k \in \mathbb{Z}$

2. Determine the exact value of each trigonometric ratio.

a) $\cos 30^\circ$
 $= \frac{\sqrt{3}}{2}$

b) $\tan(-60^\circ)$
 $= -\tan 60^\circ$
 $= -\sqrt{3}$

c) $\csc 90^\circ$
 $= \frac{1}{\sin 90^\circ}$
 $= 1$

d) $\csc(-390^\circ)$
 $= \frac{1}{\sin(-390^\circ)}$
 $= \frac{1}{\sin(-30^\circ)}$
 $= -2$

e) $\sin(-300^\circ)$
 $= \sin 60^\circ$
 $= \frac{\sqrt{3}}{2}$

f) $\sec 225^\circ$
 $= \frac{1}{\cos 225^\circ}$
 $= -\frac{1}{\cos 45^\circ}$
 $= -\sqrt{2}$

g) $\cot 495^\circ$
 $= \frac{1}{\tan 495^\circ}$
 $= \frac{1}{\tan 135^\circ}$
 $= -1$

h) $\tan 630^\circ$
 $= \tan 270^\circ$, which is undefined

3. P(2, -3) is a terminal point of angle θ in standard position.
To the nearest degree, determine possible values of θ in the domain $-360^\circ \leq \theta \leq 360^\circ$.

The terminal arm of angle θ is in Quadrant 4.

$$\tan \theta = -\frac{3}{2}, \text{ or } -1.5$$

The reference angle is: $\tan^{-1}(1.5) = 56.3099\dots^\circ$

$$\text{So, } \theta = 360^\circ - 56.3099\dots^\circ \\ = 303.6900\dots^\circ$$

Another value of θ in the given domain is: $-56.3099\dots^\circ$

To the nearest degree, θ is -56° or 304° .

4. Given $\cos \theta = \frac{1}{3}$

- a) Determine the exact values of the other trigonometric ratios for $180^\circ \leq \theta \leq 360^\circ$.

Let P(x, y) on a circle, radius r, be the terminal point of angle θ in standard position.

$$\text{Then, } \cos \theta = \frac{x}{r}$$

$$\frac{x}{r} = \frac{1}{3}, \text{ so choose } x = 1 \text{ and } r = 3$$

Use: $x^2 + y^2 = r^2$ Substitute for x and r.

$$1 + y^2 = 9$$

$$y = \pm \sqrt{8}$$

Since $\cos \theta$ is positive, for the given domain the terminal arm of θ lies in Quadrant 4 where y is negative; so $y = -\sqrt{8}$

$$\sec \theta = 3 \quad \sin \theta = -\frac{\sqrt{8}}{3} \quad \csc \theta = -\frac{3}{\sqrt{8}} \quad \tan \theta = -\sqrt{8} \quad \cot \theta = -\frac{1}{\sqrt{8}}$$

- b) To the nearest degree, determine all possible values of θ in the domain $-360^\circ \leq \theta \leq 360^\circ$.

$$\cos \theta = \frac{1}{3}, \text{ so the reference angle is: } \cos^{-1}\left(\frac{1}{3}\right) = 70.5287\dots^\circ$$

$$\text{So, } \theta = -70.5287\dots^\circ \text{ or } \theta = 360^\circ - 70.5287\dots^\circ \\ = 289.4712\dots^\circ$$

To the nearest degree, θ is -71° or 289° .

5. **Multiple Choice** Point P(-2, -1) lies on the terminal arm of angle θ in standard position. Which statement is correct?

A. $\tan \theta = \frac{1}{2}$

B. $\sin \theta = -\frac{2}{\sqrt{5}}$

C. $\cos \theta = -\frac{1}{\sqrt{5}}$

D. $\csc \theta = \frac{\sqrt{5}}{2}$

8.2

6. In terms of π , determine the length of the arc that subtends a central angle of 240° in a circle with radius 9 cm.

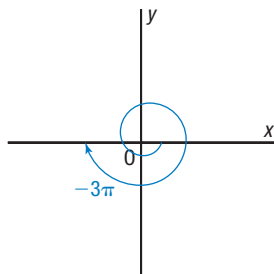
Arc length is: $\frac{240}{360}(2\pi)(9) = 12\pi$

The arc length is 12π cm.

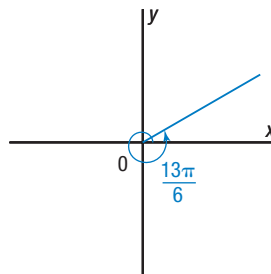
8.3

7. a) Sketch each angle θ in standard position.

i) $\theta = -3\pi$



ii) $\theta = \frac{13\pi}{6}$



- b) For each angle θ in part a

- i) Determine the measures of angles that are coterminal with θ in the domain $-4\pi \leq \theta \leq 4\pi$.

i) The measures of the angles coterminal with -3π are:
 -3π ; $-3\pi + 2\pi = -\pi$;
 $-3\pi + 4\pi = \pi$;
 and $-3\pi + 6\pi = 3\pi$
 The measures of all angles are: $-3\pi + 2\pi k, k \in \mathbb{Z}$

- ii) Write an expression for all angles that are coterminal with θ .

ii) The measures of the angles coterminal with $\frac{13\pi}{6}$ are: $\frac{13\pi}{6}$;
 $\frac{13\pi}{6} - 2\pi = \frac{\pi}{6}$;
 $\frac{13\pi}{6} - 4\pi = -\frac{11\pi}{6}$; and
 $\frac{13\pi}{6} - 6\pi = -\frac{23\pi}{6}$;
 The measures of all angles are:
 $\frac{13\pi}{6} + 2\pi k, k \in \mathbb{Z}$

8. Multiple Choice For which pair of values of θ is $\sin \theta = -\cos \theta$?

A. $\theta = \frac{\pi}{4}$ and $\theta = \frac{3\pi}{4}$

B. $\theta = \frac{3\pi}{4}$ and $\theta = \frac{5\pi}{4}$

C. $\theta = \frac{5\pi}{4}$ and $\theta = \frac{7\pi}{4}$

D. $\theta = \frac{3\pi}{4}$ and $\theta = \frac{7\pi}{4}$

9. Determine the exact value of each trigonometric ratio.

$$\begin{aligned} \text{a) } \tan \frac{\pi}{4} \\ = 1 \end{aligned}$$

$$\begin{aligned} \text{b) } \csc \left(-\frac{\pi}{6} \right) \\ = -\frac{1}{\sin \frac{\pi}{6}} \\ = -2 \end{aligned}$$

$$\begin{aligned} \text{c) } \sin \frac{11\pi}{3} \\ = -\sin \frac{\pi}{3} \\ = -\frac{\sqrt{3}}{2} \end{aligned}$$

$$\begin{aligned} \text{d) } \sin \frac{10\pi}{3} \\ = \sin \frac{4\pi}{3} \\ = -\sin \frac{\pi}{3} \\ = -\frac{\sqrt{3}}{2} \end{aligned}$$

$$\begin{aligned} \text{e) } \sec \left(-\frac{8\pi}{3} \right) \\ = \sec \left(-\frac{2\pi}{3} \right) \\ = \frac{1}{\cos \frac{2\pi}{3}} \\ = -2 \end{aligned}$$

$$\begin{aligned} \text{f) } \cot \frac{13\pi}{6} \\ = \cot \frac{\pi}{6} \\ = \frac{1}{\tan \frac{\pi}{6}} \\ = \sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{g) } \cos \left(-\frac{11\pi}{4} \right) \\ = \cos \left(-\frac{3\pi}{4} \right) \\ = \cos \frac{3\pi}{4} \\ = -\frac{1}{\sqrt{2}} \end{aligned}$$

$$\begin{aligned} \text{h) } \sec \left(-\frac{17\pi}{6} \right) \\ = \frac{1}{\cos \left(-\frac{5\pi}{6} \right)} \\ = \frac{1}{-\frac{\sqrt{3}}{2}} \\ = -\frac{2}{\sqrt{3}} \end{aligned}$$

10. $P(-5, -2)$ is a terminal point of angle θ in standard position.

a) State the exact values of all trigonometric ratios for θ .

Let the distance between the origin and P be r .

Use: $x^2 + y^2 = r^2$ Substitute: $x = -5, y = -2$

$$25 + 4 = r^2$$

$$r = \sqrt{29}$$

$$\sin \theta = -\frac{2}{\sqrt{29}} \quad \csc \theta = -\frac{\sqrt{29}}{2}$$

$$\cos \theta = -\frac{5}{\sqrt{29}} \quad \sec \theta = -\frac{\sqrt{29}}{5}$$

$$\tan \theta = \frac{2}{5} \quad \cot \theta = \frac{5}{2}$$

b) To the nearest tenth of a radian, determine all possible values of θ in the domain $-2\pi \leq \theta \leq 2\pi$.

The terminal arm of angle θ lies in Quadrant 3.

The reference angle is: $\tan^{-1} \left(\frac{2}{5} \right) = 0.3805 \dots$

$$\begin{aligned} \text{So, } \theta &= \pi + 0.3805 \dots \\ &= 3.5220 \dots \end{aligned}$$

The angle between -2π and 0 that is coterminal with 3.5220... is:

$$-2\pi + 3.5220 \dots = -2.7610 \dots$$

The possible values of θ are approximately: 3.5 and -2.7

11. a) Convert each angle to radians.

i) 450°

$$\begin{aligned} &= 450 \left(\frac{\pi}{180} \right) \\ &= \frac{5\pi}{2} \end{aligned}$$

ii) -115°

$$\begin{aligned} &= 115 \left(-\frac{\pi}{180} \right) \\ &= -\frac{23\pi}{36} \end{aligned}$$

iii) -375°

$$\begin{aligned} &= 375 \left(-\frac{\pi}{180} \right) \\ &= -\frac{25\pi}{12} \end{aligned}$$

b) Convert each angle to degrees. Give the exact answer where possible. Write the answer to the nearest degree where necessary.

i) $\frac{7\pi}{4}$

$$\begin{aligned} &= \frac{7}{4}(180^\circ) \\ &= 315^\circ \end{aligned}$$

ii) $-\frac{4\pi}{7}$

$$\begin{aligned} &= \frac{4}{7}(-180^\circ) \\ &\doteq -103^\circ \end{aligned}$$

iii) $-\frac{17\pi}{6}$

$$\begin{aligned} &= \frac{17(-180^\circ)}{6} \\ &= -510^\circ \end{aligned}$$

12. An arc of length 4.5 cm is marked on the circumference of a circle with radius 10.0 cm. What is the area of the sector of the circle formed by the arc and the radii that join the centre of the circle to the endpoints of the arc?

$$\begin{aligned} \text{Measure of central angle} &= \frac{\text{arc length}}{\text{radius}} \\ &= \frac{4.5}{10} \\ &= 0.45 \end{aligned}$$

The area of the sector is proportional to the central angle.

$$\begin{aligned} \frac{\text{Area of sector}}{\text{Area of circle}} &= \frac{0.45}{2\pi} \\ \text{Area of sector} &= \frac{0.45\pi(10)^2}{2\pi} \text{ cm}^2 \\ \text{Area of sector} &= 22.5 \text{ cm}^2 \end{aligned}$$