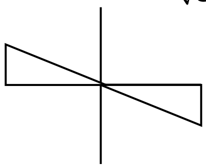
Pre-Calculus 12 Name:___________________________________

Ch. 9 Review Solutions

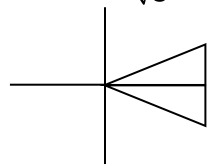
A. Solve for $0 \le x < 2\pi$:

A1. $\tan x = -\frac{1}{\sqrt{3}}$



$$x = \frac{5\pi}{6}, \frac{11\pi}{6}$$

A2. $\sec x = \frac{2}{\sqrt{3}}$



$$x = \frac{\pi}{6}, \frac{11\pi}{6}$$

B. Solve for $0 \le x < 2\pi$:

B1. $\sin^2 x - 3\sin x - 4 = 0$

$(s-4)(s+1) = 0$

→ $\sin x = 4$, $\sin x = -1$

(reject) $x = \frac{3\pi}{2}$

B2. $2\cos^2 x + \cos x = 1$

$2\cos^2 x + \cos x - 1 = 0$

$(2c-1)(c+1) = 0$

→ $\cos x = \frac{1}{2}$, $\cos x = -1$

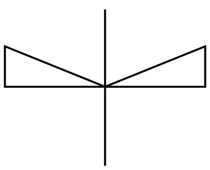
$$x = \frac{\pi}{3}, \frac{5\pi}{3}, \pi$$

C. Solve for $0 \le x < 2\pi$:

C1. $\sin 3x = \frac{\sqrt{3}}{2}$

HC by $\frac{1}{3}$

3 cycles = 6 solutions



$x = \frac{\pi}{3}\left(\frac{1}{3}\right), \frac{2\pi}{3}\left(\frac{1}{3}\right), \ldots$

$x = \frac{\pi}{9}, \frac{2\pi}{9}$

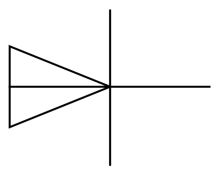
$x = \frac{7\pi}{9}, \frac{8\pi}{9}$

$x = \frac{13\pi}{9}, \frac{14\pi}{9}$

C2. $\cos 2x = -\frac{1}{\sqrt{2}}$

HC by $\frac{1}{2}$

2 cycles = 4 solutions



$x = \frac{3\pi}{4}\left(\frac{1}{2}\right), \frac{5\pi}{4}\left(\frac{1}{2}\right), \ldots$

$x = \frac{3\pi}{8}, \frac{5\pi}{8}$

$x = \frac{11\pi}{8}, \frac{13\pi}{8}$

D1. Verify the identity $\frac{\cot x}{\csc x \cos x} = 1$ for $\theta = \frac{\pi}{6}$.

$$\frac{\cot\frac{\pi}{6}}{\csc\frac{\pi}{6}\cos\frac{\pi}{6}} = 1$$

$$\frac{\frac{\sqrt{3}}{1}}{\left(\frac{2}{1}\right)\left(\frac{\sqrt{3}}{2}\right)} = 1 \quad \rightarrow \quad \frac{\sqrt{3}}{\sqrt{3}} = 1 \quad \rightarrow \quad 1 = 1$$

E1. Determine the non-permissible values for the identity $\sin x+\cos x\cot x=\csc x$.

$$\sin x+\cos x\frac{\cos x}{\sin x}=\frac{1}{\sin x} \quad \rightarrow \quad \begin{array}{l} \sin x\neq 0 \\ \therefore x\neq 0,\pi,\ldots \\ x\neq n\pi\ (n\notin \mathbb{Z}) \end{array}$$

F1. Prove the identity: $\dfrac{\cot x}{\csc x\cos x}=1$

LS	RS
$\dfrac{\cot x}{\csc x\cos x}$	1
$=\dfrac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x}\cos x}$	
$=\dfrac{\cos x}{\sin x}\left(\dfrac{\sin x}{\cos x}\right)$	
$=1$	

LS = RS

F2. Prove the identity: $\dfrac{\csc x\sec x}{\cot x}=\sec^2 x$

LS	RS
$\dfrac{\csc x\sec x}{\cot x}$	$\sec^2 x$
$=\dfrac{\frac{1}{\sin x}\left(\frac{1}{\cos x}\right)}{\frac{\cos x}{\sin x}}$	$=\dfrac{1}{\cos^2 x}$
$=\dfrac{1}{\sin x\cos x}\left(\dfrac{\sin x}{\cos x}\right)$	
$=\dfrac{1}{\cos^2 x}$	

LS = RS

F3. Prove the identity: $\dfrac{\csc^2 x-\cot^2 x}{\cos x}=\sec x$

LS	RS
$\dfrac{\csc^2 x-\cot^2 x}{\cos x}$	$\sec x$
$=\dfrac{\frac{1}{\sin^2 x}-\frac{\cos^2 x}{\sin^2 x}}{\cos x}$	$=\dfrac{1}{\cos x}$
$=\dfrac{\frac{1-\cos^2 x}{\sin^2 x}}{\cos x}$	
$=\dfrac{\frac{\sin^2 x}{\sin^2 x}}{\cos x}$	
$=\dfrac{1}{\cos x}$	

LS = RS

F4. Prove the identity: $\sin x+\cos x\cot x=\csc x$

LS	RS
$\sin x+\cos x\cot x$	$\csc x$
$=\sin x+\cos x\dfrac{\cos x}{\sin x}$	$=\dfrac{1}{\sin x}$
$=\dfrac{\sin^2 x}{\sin x}+\dfrac{\cos^2 x}{\sin x}$	
$=\dfrac{\sin^2 x+\cos^2 x}{\sin x}$	
$=\dfrac{1}{\sin x}$	

LS = RS

G. Solve for $0 \le x < 2\pi$:

G1. $\cos^2 x + 3\sin x - 3 = 0$

$1 - \sin^2 x + 3\sin x - 3 = 0$

$0 = \sin^2 x - 3\sin x + 2$

$0 = (\sin x - 2)(\sin x - 1)$

$\rightarrow \sin x = 2, \sin x = 1$

$\boxed{x = \frac{\pi}{2}}$

G2. $2\cos x + 1 - \sin^2 x = 3$

$2\cos x + \cos^2 x = 3$

$\cos^2 x + 2\cos x - 3 = 0$

$(\cos x + 3)(\cos x - 1) = 0$

$\rightarrow \cos x = -3, \cos x = 1$

$\boxed{x = 0}$

H. Write as a single trigonometric function in simplest form. Evaluate if possible.

H1. $\cos\frac{\pi}{2}\cos\frac{\pi}{3} - \sin\frac{\pi}{2}\sin\frac{\pi}{3}$

$= \cos\left(\frac{\pi}{2} + \frac{\pi}{3}\right) = \cos\left(\frac{3\pi}{6} + \frac{2\pi}{6}\right)$

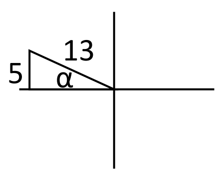
$\boxed{= \cos\left(\frac{5\pi}{6}\right) = \frac{-\sqrt{3}}{2}}$

H2. $2\cos^2 2x - 1$

$= \cos 2(2x)$

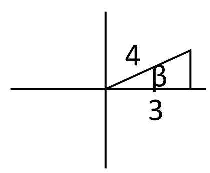
$\boxed{= \cos 4x}$

I. Angle α is in standard position with its terminal arm in quadrant II and $\sin\alpha = \frac{5}{13}$. Angle β is in standard position with its terminal arm in quadrant I and $\cos\beta = \frac{3}{4}$. Determine the exact value of:



$a^2 + 5^2 = 13^2$

$a = 12$



$3^2 + b^2 = 4^2$

$b = \sqrt{7}$

I1. $\cos(\alpha - \beta)$

$= \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$= \left(\frac{-12}{13}\right)\left(\frac{3}{4}\right) + \left(\frac{5}{13}\right)\left(\frac{\sqrt{7}}{4}\right)$

$\boxed{= \frac{-36 + 5\sqrt{7}}{52}}$

I2. $\sin 2\beta$

$= 2\sin\beta\cos\beta$

$= 2\left(\frac{\sqrt{7}}{4}\right)\left(\frac{3}{4}\right)$

$\boxed{= \frac{6\sqrt{7}}{16} = \frac{3\sqrt{7}}{8}}$

I3. $\cos 2\alpha$

$= \cos^2\alpha - \sin^2\alpha$

$= \left(\frac{-12}{13}\right)^2 - \left(\frac{5}{13}\right)^2$

$= \frac{144 - 25}{169} = \boxed{\frac{119}{169}}$

J1. Determine the exact value of $\sin\frac{7\pi}{12}$ by evaluating $\sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$.

$= \sin\frac{\pi}{3}\cos\frac{\pi}{4} + \cos\frac{\pi}{3}\sin\frac{\pi}{4}$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$

$\boxed{= \frac{\sqrt{3} + 1}{2\sqrt{2}}}$

J2. Prove that $\sin(\pi-\theta)=\sin\theta$.

LS	RS
$\sin\pi\cos\theta-\cos\pi\sin\theta$	
$=0\bullet\cos\theta-(-1)\sin\theta$	$\sin\theta$
$=0+\sin\theta$	
$=\sin\theta$	

LS = RS

F5. Prove the identity: $\csc 2x-\cot 2x=\tan x$

LS	RS
$\csc 2x-\cot 2x$	
$=\frac{1}{\sin 2x}-\frac{\cos 2x}{\sin 2x}$	
$=\frac{1-\cos 2x}{\sin 2x}$	$\tan x$
$=\frac{1-(1-2\sin^2 x)}{2\sin x\cos x}$	$=\frac{\sin x}{\cos x}$
$=\frac{1-1+2\sin^2 x}{2\sin x\cos x}$	
$=\frac{2\sin^2 x}{2\sin x\cos x}$	
$=\frac{\sin x}{\cos x}$	

LS = RS

G3. Solve $\cos 2x+\sin x=1$ for $0\le x<2\pi$.

$1-2\sin^2 x+\sin x=1$

$0=2\sin^2 x-\sin x$

$0=\sin x(2\sin x-1)$

$\rightarrow \sin x=0, \sin x=\frac{1}{2}$

$$x=0,\pi,\frac{\pi}{6},\frac{5\pi}{6}$$