Pre-Calculus 12 Name:______________________

Ch. 7 Extra Practice

A. Two solutions to the equation $\sin 3x + 2 = 3\sin 3x + 1$ are x = 0.17 and 0.87. Write a general solution to represent all the solutions to the equation.

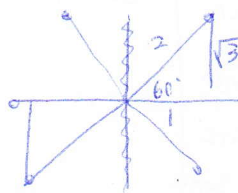
period $= 2\pi\left(\frac{1}{3}\right)$

$= \frac{2\pi}{3}$

$\boxed{x = 0.17 + \frac{2\pi}{3}n \quad (n \in \mathbb{Z}) \qquad x = 0.87 + \frac{2\pi}{3}n}$

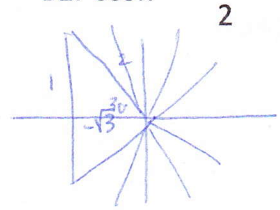
B. Solve for $0 \le x < 2\pi$:

B1. $\tan x = \sqrt{3}$



$\boxed{x = \frac{\pi}{3}, \frac{4\pi}{3}}$

B2. $\cos x = -\frac{\sqrt{3}}{2}$



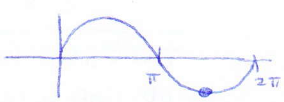
$\boxed{x = \frac{5\pi}{6}, \frac{7\pi}{6}}$

C. Solve for $0 \le x < 2\pi$:

C1. $\sin^2 x - 2\sin x - 3 = 0$

$(\sin x - 3)(\sin x + 1) = 0$

$\cancel{\sin x = 3} \qquad \sin x = -1$

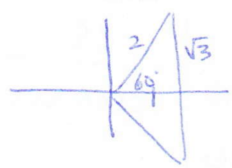


$\boxed{x = \frac{3\pi}{2}}$

C2. $2\cos^2 x - 5\cos x + 2 = 0$

$(2\cos x - 1)(\cos x - 2) = 0$

$\cos x = \frac{1}{2} \qquad \cancel{\cos x = 2}$



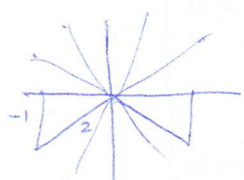
$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$

D. Solve for $0 \le x < 2\pi$:

D1. $\sin 2x = -\frac{1}{2}$

HC $\frac{1}{2}$

period $= \pi = \frac{12\pi}{12}$



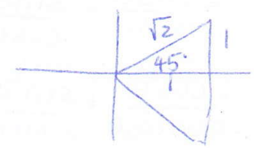
$\frac{7\pi}{6}\left(\frac{1}{2}\right) = \boxed{\frac{7\pi}{12}} + \frac{12\pi}{12} = \boxed{\frac{19\pi}{12}}$

$\frac{11\pi}{6}\left(\frac{1}{2}\right) = \boxed{\frac{11\pi}{12}} + \frac{12\pi}{12} = \boxed{\frac{23\pi}{12}}$

D2. $\cos 3x = \frac{1}{\sqrt{2}}$

HC $\frac{1}{3}$

period $= \frac{2\pi}{3} = \frac{8\pi}{12}$



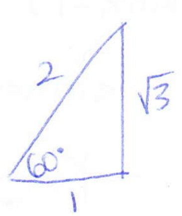
$\frac{3\pi}{4}$

$\frac{\pi}{4}\left(\frac{1}{3}\right) = \boxed{\frac{\pi}{12}} + \frac{8\pi}{12} = \boxed{\frac{9\pi}{12}} + \frac{8\pi}{12} = \boxed{\frac{17\pi}{12}}$

$\frac{7\pi}{4}\left(\frac{1}{3}\right) = \boxed{\frac{7\pi}{12}} + \frac{8\pi}{12} = \boxed{\frac{15\pi}{12}} + \frac{8\pi}{12} = \boxed{\frac{23\pi}{12}}$

$\frac{5\pi}{4}$

E1. Verify the identity $\frac{\tan\theta}{\sin\theta} = \sec\theta$ for $\theta = \frac{\pi}{3}$ ← 60°



$$\frac{\left(\frac{\sqrt{3}}{1}\right)}{\left(\frac{\sqrt{3}}{2}\right)} \stackrel{?}{=} \frac{2}{1}$$

$$\Rightarrow \left(\frac{\sqrt{3}}{1}\right)\left(\frac{2}{\sqrt{3}}\right) \stackrel{?}{=} \frac{2}{1}$$

$$\frac{2}{1} = \frac{2}{1}$$

$$2 = 2 \checkmark$$

F1. Determine the non-permissible values for the identity $\tan\theta\sin\theta+\cos\theta=\sec\theta$.

$\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ $\quad\cos\theta\neq 0$

$\sec\theta=\dfrac{1}{\cos\theta}$

$\Rightarrow \boxed{\theta\neq\frac{\pi}{2}+n\pi\ (n\in\mathbb{Z})}$

G1. Prove the identity: $\dfrac{\tan\theta}{\sin\theta}=\sec\theta$

LS	RS
$\dfrac{\tan\theta}{\sin\theta}$	$\sec\theta$
$=\dfrac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta}{1}}$	
$=\dfrac{\sin\theta}{\cos\theta}\times\dfrac{1}{\sin\theta}$	
$=\dfrac{1}{\cos\theta}$	LS=RS
$=\sec\theta$	

G2. Prove the identity: $\dfrac{\tan\theta}{\sec\theta}+\dfrac{\cot\theta}{\csc\theta}=\sin\theta+\cos\theta$

LS	RS
$\dfrac{\tan\theta}{\sec\theta}+\dfrac{\cot\theta}{\csc\theta}$	$\sin\theta+\cos\theta$
$=\dfrac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}}+\dfrac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}}$	
$=\dfrac{\sin\theta}{\cancel{\cos\theta}}\times\dfrac{\cos\theta}{1}+\dfrac{\cos\theta}{\cancel{\sin\theta}}\times\dfrac{\sin\theta}{1}$	
$=\sin\theta+\cos\theta$	
LS=RS	

G3. Prove the identity: $\dfrac{1}{\tan\theta}+\tan\theta=\csc\theta\sec\theta$

LS	RS
$\dfrac{1}{\tan\theta}+\tan\theta$	$\csc\theta\sec\theta$
$=\cot\theta+\tan\theta$	
$=\dfrac{\cos\theta}{\sin\theta}+\dfrac{\sin\theta}{\cos\theta}$	
$=\dfrac{\cos^2\theta}{\sin\theta\cos\theta}+\dfrac{\sin^2\theta}{\sin\theta\cos\theta}$	
$=\dfrac{\cos^2\theta+\sin^2\theta}{\sin\theta\cos\theta}$	
$=\dfrac{1}{\sin\theta\cos\theta}$	LS=RS
$=\csc\theta\sec\theta$	

G4. Prove the identity: $\tan\theta\sin\theta+\cos\theta=\sec\theta$

LS	RS
$\tan\theta\sin\theta+\cos\theta$	$\sec\theta$
$=\dfrac{\sin\theta\cdot\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{1}$	
$=\dfrac{\sin^2\theta}{\cos\theta}+\dfrac{\cos^2\theta}{\cos\theta}$	
$=\dfrac{\sin^2\theta+\cos^2\theta}{\cos\theta}$	
$=\dfrac{1}{\cos\theta}$	
$=\sec\theta$	
LS=RS	

H. Solve for $0\le x<2\pi$:

H1. $\cos x+\sec x=2$

$\left(\cos x+\dfrac{1}{\cos x}=2\right)\cos x$

$\cos^2 x+1=2\cos x$

$\cos^2 x-2\cos x+1=0$

$(\cos x-1)(\cos x-1)=0$

$\cos x=1$ $\quad\cos x=1$

$\boxed{x=0}$

H2. $2\cos^2 x=\sin x+1$

$2(1-\sin^2 x)=\sin x+1$

$2-2\sin^2 x=\sin x+1$

$0=2\sin^2 x+\sin x-1$

$0=(2\sin x-1)(\sin x+1)$

$\sin x=\frac{1}{2}$ $\quad\sin x=-1$

$\boxed{x=\frac{\pi}{6},\frac{5\pi}{6},\frac{3\pi}{2}}$