Pre-Calcuclus 12 **Ch. 7 Part II Review** **Name:** ________________

A1. Write $\log_a(b+2)=c$ in exponential form. $\boxed{a^c = b+2}$

A2. Write $y-1=3^{x+2}$ in logarithmic form. $\boxed{\log_3(y-1) = x+2}$

B1. Evaluate: $\log_{\sqrt{5}} 125$ $\quad \sqrt{5}^{\square} = 125 \Rightarrow 5^{\frac{1}{2}x} = 5^3 \quad \boxed{x=6}$

B2. Evaluate: $2\log_4 16 + \frac{1}{3}\log_2\left(\frac{1}{8}\right)$ $= 2(2) + \frac{1}{3}(-3) = 4 - 1 = \boxed{3}$

C. Use benchmarks to estimate the value of $\log_2 60$.

5 ——— 6
32 ——— 60 64

$\boxed{\sim 5.9}$

D1. Write $3\log a + \frac{1}{2}\log b - \frac{1}{4}\log c$ as a single logarithm.

$\Rightarrow \log a^3 + \log b^{\frac{1}{2}} - \log c^{\frac{1}{4}} = \boxed{\log \frac{a^3\sqrt{b}}{\sqrt[4]{c}}}$

D2. Evaluate $\log_3\sqrt{54} - \log_3\sqrt{6}$. $= \log_3 \frac{\sqrt{54}}{\sqrt{6}} \Rightarrow \log_3\sqrt{9} \Rightarrow \log_3 3 = \boxed{1}$

D3. Write $\log\left(\frac{a^2}{bc^3}\right)$ in terms of $\log a$, $\log b$, and/or $\log c$.

$= \log a^2 - \log b - \log c^3 = \boxed{2\log a - \log b - 3\log c}$

D4. If $\log_3 x = 2$ and $\log_3 y = 5$, evaluate $\log_3\left(\frac{3x^2}{y}\right) \Rightarrow \log_3 3 + \log_3 x^2 - \log_3 y$

$= \log_3 3 + 2\log_3 x - \log_3 y$

$= 1 + 2(2) - (5) = \boxed{0}$

E. Evaluate to 3 decimal places: $\log_4 75$. $= \boxed{3.114}$

F. Sketch the graph of $y = \log_2(x+2)+1$.

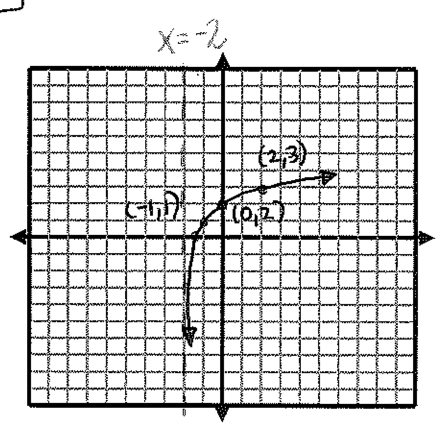
left 2
up 1

$y=2^x$

x	y
-2	1/4
-1	1/2
0	1
1	2
2	4

$\Rightarrow$ $y=\log_2 x$

	x	y	
$-1\frac{3}{4}$	1/4	-2	-1
$-1\frac{1}{2}$	1/2	-1	0
-1	1	0	1
0	2	1	2
2	4	2	3



G. Determine the domain, range, equation of the asymptote, and intercepts of the graph in F.

Dom: $x > -2$ $\quad$ asymptote: $x = -2$ $\quad$ x-int $= (-1\frac{1}{2}, 0)$

Range: $y \in \mathbb{R}$ $\quad$ y-int $= (0,2)$

H1. Solve for x to 3 decimal places: $5^{x-3} = 2^{x+1}$.

$(x-3)\log 5 = (x+1)\log 2$

$x\log 5 - 3\log 5 = x\log 2 + \log 2$

$x\log 5 - x\log 2 = \log 2 + 3\log 5$

$x \log \frac{5}{2} = \log(2\cdot 5^3)$

$x = \frac{\log 250}{\log \frac{5}{2}}$ $\quad \boxed{x = 6.026}$

H2. Solve for x to 3 decimal places: $2^x = 3\left(4^{x+1}\right)$.

$$\log 2^x = \log 3(4^{x+1})$$
$$x\log 2 = \log 3 + \log 4^{x+1}$$
$$x\log 2 = \log 3 + (x+1)\log 4$$
$$x\log 2 = \log 3 + x\log 4 + \log 4$$
$$x\log 2 - x\log 4 = \log 3 + \log 4$$
$$x\log \tfrac{2}{4} = \log(3\cdot 4)$$
$$x = \frac{\log 12}{\log \frac{2}{4}} \Rightarrow \boxed{x = -3.585}$$

I1. Solve for x: $\log(x+11)+\log x = \log(x+1)+\log 6$.

$$\log \underbrace{(x+11)(x)} = \log \underbrace{(x+1)(6)}$$
$$x^2+11x = 6x+6$$
$$x^2+5x-6=0$$
$$(x+6)(x-1)=0$$
$$\cancel{x=-6},\ \boxed{x=1}$$

extraneous root

I2. Solve for x: $\log_2(x+2)+\log_2 x = 3$.

$$\log_2 \underbrace{(x+2)(x)} = \log_2 8$$
$$x^2+2x = 8$$
$$x^2+2x-8=0$$
$$(x+4)(x-2)=0$$
$$\cancel{x=-4},\ \boxed{x=2}$$

extraneous root

J1. You invest \$5000 in an account with a fixed interest rate of 3%/annum, compounded semi-annually. How long will it take for the investment to double? $\div 2$ ↳ 1.5% ↳ period = ½ year

⇒ rate = 101.5%

$$\frac{10000}{5000} = \frac{5000}{5000}(1.015)^{t/\frac{1}{2}}$$

★ $\boxed{\dfrac{t}{\frac{1}{2}} = t\cdot\dfrac{2}{1} = 2t}$

$$2 = (1.015)^{2t}$$
$$\log 2 = \log 1.015^{2t}$$
$$\log 2 = 2t\log 1.015$$
$$\frac{\log 2}{\log 1.015} = 2t$$
$$t = \frac{\log 2}{\log 1.015} \div 2$$
$$\boxed{t = 23.3 \text{ years}}$$

J2. Parents plan to invest money for their newborn son so that he has \$20 000 available for his education on his 18[th] birthday. Assuming a growth rate of 6% per year, compounded monthly, how much will they need to invest today? $\div 12$ ↳ 0.5% ⇒ 100.5% ↳ period = 1/12 year

$$20000 = P_0(1.005)^{18/\frac{1}{12}}$$
$$\frac{20000}{1.005^{216}} = \frac{P_0(\cancel{1.005})^{216}}{\cancel{1.005^{216}}}$$
$$\boxed{P_0 = \$6810.21}$$

J3. A radioactive isotope has a half-life of approximately 12 weeks. How much of a sample of 30 grams would remain after 50 weeks? (Round to the nearest hundredth of a gram)

$$P = 30(0.5)^{\frac{50}{12}} \Rightarrow \boxed{1.67\text{ g}}$$

K1. How many times as intense as a 6.3 magnitude earthquake is an 8.4 magnitude earthquake?

$$\frac{10^{8.4}}{10^{6.3}} = 10^{8.4-6.3} = 10^{2.1} = \boxed{125.9 \text{ times}}$$

K2. How many times louder is a referee's whistle (125 dB) than a flute (89 dB)?

$$\frac{10^{125/10}}{10^{89/10}} = 10^{\frac{125}{10}-\frac{89}{10}} = 10^{\frac{36}{10}} = 10^{3.6} = \boxed{3981.1 \text{ times}}$$

L ~~K3~~. Tomato juice has a pH level of 4.0 Determine the pH level of a solution that is 5 times more acidic.

$$\frac{10^4}{10^x} = 5 \Rightarrow 10^{4-x} = 5 \rightarrow \log 10^{4-x} = \log 5 \Rightarrow 4-x\,\underbrace{\log 10}_{=1} = \log 5$$
$$4-x = \log 5$$
$$x = 4-\log 5$$
$$\boxed{x = 3.3}$$

lower

M1. L1. Use natural logarithms to solve the exponential equation $\frac{5e^{x-2}}{5} = \frac{120}{5}$ to 3 decimal places:

$$e^{x-2} = 24$$
$$\ln e^{x-2} = \ln 24$$
$$x-2\,\underbrace{\ln e}_{=1} = \ln 24$$
$$x-2 = \ln 24$$
$$x = \ln 24 + 2$$
$$\boxed{x = 5.178}$$

M2. Solve the following equation: $\ln(x+3)+\ln 3 = \ln(x^2-1)$

$$\ln \underbrace{(x+3)(3)} = \ln \underbrace{(x^2-1)}$$
$$3x+9 = x^2-1$$
$$0 = x^2-3x-10$$
$$0 = (x-5)(x+2)$$
$$\boxed{x=5,\ x=-2}$$

no extraneous roots!

Answers:

A1. $a^c = b+2$ A2. $\log_3(y-1) = x+2$ B1. 6 B2. 3 C. approx 5.9 D1. $\log\frac{a^3\sqrt{b}}{\sqrt[4]{c}}$ D2. 1

D3. 2loga – logb – 3logc D4. 0 E. 3.114 F. translate 2 units left and 1 unit up.

G. Domain: $x > -2$, Range $y \in \mathbb{R}$, asymptote: $x = -2$, x-int: –1.5, y-int: 2 H1. 6.026 H2. –3.585

I1. x = 1 I2. x = 2 J1. 23.3 years J2.\$6810.21 J3. 1.67 grams

K1. 125.9 times K2. 3981 times L ~~K3~~. pH 3.3 M1 ~~L1~~. x = 5.178 M2 ~~L2~~. x = 5, x = –2