Pre-Calculus 12 Name: ____________________

9.6 – Double-Angle Identities Block: ________

$$\sin 2x = \sin(x+x) = \sin x\cos x + \cos x \sin x = 2\sin x \cos x$$

$$\cos 2x = \cos(x+x) = \cos x \cos x - \sin x \sin x = \cos^2 x - \sin^2 x$$

<u>Note:</u> using the Pythagorean identity, we can substitute $1-\sin^2 x = \cos^2 x$ or $1-\cos^2 x = \sin^2 x$ to create 2 other identities for $\cos 2x$.

$$\tan 2x = \tan(x+x) = \frac{\tan x + \tan x}{1-\tan x \tan x} = \frac{2\tan x}{1-\tan^2 x}$$

$\sin 2\theta = 2\sin\theta\cos\theta$	$\cos 2\theta = \cos^2\theta - \sin^2\theta$
	$= 1 - 2\sin^2\theta$
$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$	$= 2\cos^2\theta - 1$

1. Write each expression as a single trig ratio.

a) $\cos^2\frac{\pi}{4} - \sin^2\frac{\pi}{4} = \cos\left(2\cdot\frac{\pi}{4}\right) = \boxed{\cos\frac{\pi}{2}}$

b) $\dfrac{2\tan\frac{\pi}{6}}{1-\tan^2\frac{\pi}{6}} = \tan\left(2\cdot\frac{\pi}{6}\right) = \boxed{\tan\frac{\pi}{3}}$

c) $3 - 6\sin^2 3 = 3(1-2\sin^2 3) = 3(\cos 2\cdot 3) = \boxed{3\cos 6}$

GCF=3

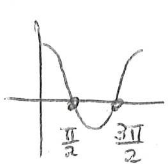
2. Solve the equation $\sin 2x + \cos x = 0$ for $0 \le x < 2\pi$.

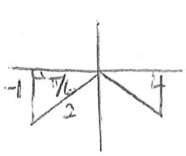
$2\sin x\cos x + \cos x = 0$

$\cos x(2\sin x + 1) = 0$

$\cos x = 0$ $\sin x = -\frac{1}{2}$

$\boxed{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}}$





* 3 options for $\cos 2\theta$ choose $2\cos^2\theta - 1$, so the -1 and $+1$ cancel and $= 0$.

3. Prove the following identities.

a) $\cot\theta \csc 2\theta = \dfrac{1}{2\sin^2\theta}$

LS	RS
$\cot\theta \csc 2\theta$	$\dfrac{1}{2\sin^2\theta}$
$= \dfrac{\cos\theta}{\sin\theta} \cdot \dfrac{1}{\sin 2\theta}$	
$= \dfrac{\cos\theta}{\sin\theta} \cdot \dfrac{1}{2\sin\theta\cos\theta}$	
$= \dfrac{1}{2\sin^2\theta}$	

LS = RS

b) $\cot\theta = \dfrac{\cos 2\theta + 1}{\sin 2\theta}$

LS	RS
$\cot\theta$	$\dfrac{\cos 2\theta^{*} + 1}{\sin 2\theta}$
	$= \dfrac{2\cos^2\theta - 1 + 1}{2\sin\theta\cos\theta}$
	$= \dfrac{2\cos^2\theta}{2\sin\theta\cos\theta}$
	$= \dfrac{\cos\theta}{\sin\theta}$
	$= \cot\theta$

LS = RS

4. Given angle Θ is in Quadrant IV and $\cos\theta = \dfrac{2}{5}$, determine:

a) $\cos 2\theta$

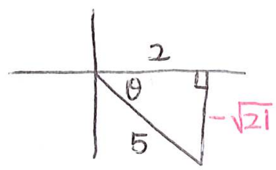
$= 2\cos^2\theta - 1$

$= 2\left(\frac{2}{5}\right)^2 - 1$

$= 2\left(\frac{4}{25}\right) - 1$

$= \frac{8}{25} - \frac{25}{25}$

$= \boxed{\frac{-17}{25}}$



$2^2 + y^2 = 5^2$

$y^2 = 25 - 4$

$y^2 = \pm\sqrt{21}$

b) $\sin 2\theta$

$= 2\sin\theta\cos\theta$

$= 2\left(\frac{-\sqrt{21}}{5}\right)\left(\frac{2}{5}\right)$

$= \boxed{\frac{-4\sqrt{21}}{25}}$

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p. ~~658~~ # 4-16 and MC # 1,2