Pre-Calculus 12 Name: ______________________

9.5 – The Sum and Difference Identities Block: __________

$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$	$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$
$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$	$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$
$\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$	$\tan(\alpha-\beta)=\dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

1. Simplify by writing each expression as a single trig function.

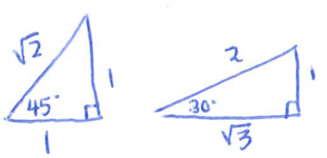
a) $\sin 8x\cos 3x-\cos 8x\sin 3x \;=\; \sin(8x-3x) = \boxed{\sin 5x}$

b) $\dfrac{\tan\frac{\pi}{6}+\tan\frac{\pi}{12}}{1-\tan\frac{\pi}{6}\tan\frac{\pi}{12}} = \tan\left(\frac{\pi}{6}+\frac{\pi}{12}\right) = \tan\left(\frac{2\pi}{12}+\frac{\pi}{12}\right) = \tan\left(\frac{3\pi}{12}\right) = \boxed{\tan\frac{\pi}{4}}$

2. Determine the exact value of cos 75° by evaluating cos (45° + 30°).

$\cos 75^\circ = \cos(45^\circ+30^\circ) = \cos 45^\circ\cos 30^\circ - \sin 45^\circ \sin 30^\circ$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$



$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$

$= \boxed{\frac{\sqrt{3}-1}{2\sqrt{2}}}$

3. Solve for x for $0 \le x < 2\pi$: $\cos 4x\cos x+\sin 4x\sin x=0$

$\Rightarrow \cos(4x-x)=0$

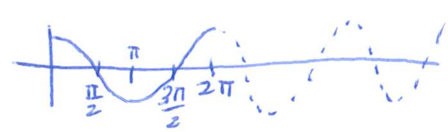
$\boxed{\cos 3x = 0}$

→ 3 cycles

→ 6 solutions

→ HC by 1/3

→ period $= \frac{2\pi}{3} = \frac{4\pi}{6}$



$x = \left[\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2}\right] \times \frac{1}{3}$

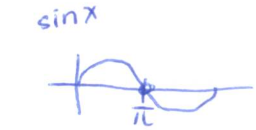
$x = \frac{\pi}{6}, \frac{3\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{9\pi}{6}, \frac{11\pi}{6}$

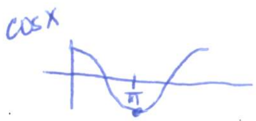
$\hookrightarrow \frac{\pi}{2}$ $\qquad \hookrightarrow \frac{3\pi}{2}$

4. Prove $\sin(\pi - x) = \sin x$.

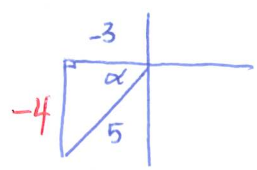
LS	RS
$\sin(\pi - x)$	$\sin x$
$= \sin\pi\cos x - \cos\pi\sin x$	
$= (0)\cos x - (-1)\sin x$	
$= 0 + \sin x$	
$= \sin x$	

LS = RS

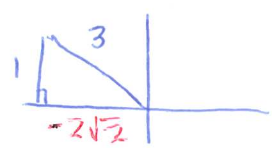




5. Given angle α is in Quadrant III with $\cos\alpha = -\frac{3}{5}$, and angle β is in Quadrant II with $\sin\beta = \frac{1}{3}$, determine sin(α + β).



$a^2 + (-3)^2 = 5^2$
$a^2 + 9 = 25$
$a^2 = 25 - 9$
$a^2 = 16$
$a = \pm 4$



$a^2 + 1^2 = 3^2$
$a^2 = 9 - 1$
$a^2 = 8$
$a = \pm\sqrt{8}$
$= \pm 2\sqrt{2}$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$= \left(\frac{-4}{5}\right)\left(\frac{-2\sqrt{2}}{3}\right) + \left(\frac{-3}{5}\right)\left(\frac{1}{3}\right)$$
$$= \frac{8\sqrt{2}}{15} - \frac{3}{15} \Rightarrow \boxed{\frac{8\sqrt{2} - 3}{15}}$$

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Note: solve #10, 13 for $0 \le x < 2\pi$