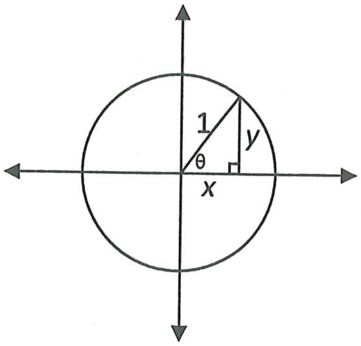
Pre-Calculus 12

9.4 – The Pythagorean Identities

Name: ____________________

Block: ________

Recall: In the unit circle,



$$\sin\theta = \frac{opp}{hyp} = \frac{y}{1} = y$$

$$\cos\theta = \frac{adj}{hyp} = \frac{x}{1} = x$$

The Pythagorean theorem states $x^2 + y^2 = 1 \Rightarrow \cos^2\theta + \sin^2\theta = 1$ or $\sin^2\theta + \cos^2\theta = 1$.

If we divide the whole equation by $\sin^2\theta$, or $\cos^2\theta$, we'll get the other Pythagorean Identities.

Note:

We can rearrange these identities!

$\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \cos^2\theta = 1 - \sin^2\theta$

Pythagorean Identities:

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$\star\ \frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$

$\Rightarrow 1 + \cot^2\theta = \csc^2\theta$

$\star\ \frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

$\Rightarrow \tan^2\theta + 1 = \sec^2\theta$

1. Provide properly formatted proofs for the following identities.

a) $\cot^3\theta = \cot\theta\csc^2\theta - \cot\theta$

LS	RS
$\cot^3\theta$	$\cot\theta\csc^2\theta - \cot\theta$
	$= \cot\theta(1+\cot^2\theta) - \cot\theta$
	$= \cot\theta + \cot^3\theta - \cot\theta$
	$= \cot^3\theta$

$\star\ \csc^2\theta = 1 + \cot^2\theta$

LS = RS

b) $\cot\theta + \tan\theta = \csc\theta\sec\theta$

LS	RS
$\cot\theta + \tan\theta$	$\csc\theta\sec\theta$
$= \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$	
$= \frac{\cos^2\theta}{\sin\theta\cos\theta} + \frac{\sin^2\theta}{\sin\theta\cos\theta}$	
$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}$	
$= \frac{1}{\sin\theta\cos\theta}$	
$= \csc\theta\sec\theta$	

Find a common denominator

Pythagorean identity

$\sin^2\theta + \cos^2\theta = 1$

LS = RS

c) $\dfrac{1-\cos\theta}{\sin\theta}=\dfrac{\sin\theta}{1+\cos\theta}$

LS	RS
$\dfrac{1-\cos\theta}{\sin\theta}$	$\dfrac{\sin\theta}{1+\cos\theta}\dfrac{(1-\cos\theta)}{(1-\cos\theta)}$ ← create a difference of squares
	$=\dfrac{\sin\theta(1-\cos\theta)}{1-\cos\theta+\cos\theta-\cos^2\theta}$
	$=\dfrac{\sin\theta(1-\cos\theta)}{1-\cos^2\theta}$ ← $\sin^2\theta+\cos^2\theta=1 \Rightarrow \sin^2\theta=1-\cos^2\theta$
	$=\dfrac{\cancel{\sin\theta}(1-\cos\theta)}{\sin^{\cancel{2}}\theta}$
LS=RS	$=\dfrac{1-\cos\theta}{\sin\theta}$

d) $\dfrac{1}{1-\cos\theta}+\dfrac{1}{1+\cos\theta}=2\csc^2\theta$

LS	RS
$\dfrac{(1+\cos\theta)}{(1+\cos\theta)}\dfrac{1}{1-\cos\theta}+\dfrac{1}{1+\cos\theta}\dfrac{(1-\cos\theta)}{(1-\cos\theta)}$ ← common denominator	$2\csc^2\theta$
$=\dfrac{1+\cos\theta}{1-\cos^2\theta}+\dfrac{1-\cos\theta}{1-\cos^2\theta}$	
$=\dfrac{1+\cos\theta+1-\cos\theta}{1-\cos^2\theta}$ ← (as above) $\sin^2\theta=1-\cos^2\theta$	
$=\dfrac{2}{\sin^2\theta}$	
$=2\csc^2\theta$	LS=RS

2. Solve $3-3\cos x-2\sin^2 x=0$ for $0\le x<2\pi$.

* can't solve with cos x and sin x!
substitute $\sin^2 x=1-\cos^2 x$

$3-3\cos x-2(1-\cos^2 x)=0$

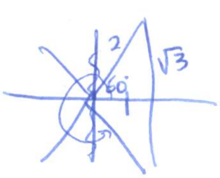
$3-3\cos x-2+2\cos^2 x=0$

$2\cos^2 x-3\cos x+1=0$

$(2\cos x-1)(\cos x-1)=0$

$\cos x=\frac{1}{2}$ $\qquad$ $\cos x=1$

$\boxed{x=0}$



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$\boxed{x=\frac{\pi}{3},\frac{5\pi}{3}}$

$\boxed{x=0,\frac{\pi}{3},\frac{5\pi}{3}}$