Pre-Calculus 12 Name: ____________________

9.3 – Reciprocal and Quotient Identities Block: ________

A **trigonometric identity** is a statement that relates trig ratios, and is true for all values of the variable for which the trig ratios are defined.

e.g. $\sin\theta\csc\theta = 1$ is an identity because it is true for all values of θ (except where $\csc\theta$ is undefined)

$\sin\theta\sec\theta = 1$ is not an identity because it is only true for $\theta = \frac{\pi}{4}, \frac{5\pi}{4}$.

A trig identity can be **verified** by substituting a value for the variable. This shows that an identity is true for specific values, but does not prove the identity is true for all values.

To **prove** an identity is true, it must be shown that one side of the equation is equal to the other, or that both sides are equal to the same expression.

Reciprocal Identities:	$\csc\theta = \frac{1}{\sin\theta}$	$\sec\theta = \frac{1}{\cos\theta}$	$\cot\theta = \frac{1}{\tan\theta}$
Quotient Identities:	$\tan\theta = \frac{\sin\theta}{\cos\theta}$ ★	$\cot\theta = \frac{\cos\theta}{\sin\theta}$	

★ Proof

$\sin\theta = \frac{opp}{hyp}$

$\cos\theta = \frac{adj}{hyp}$

$\frac{\sin\theta}{\cos\theta} = \frac{\frac{opp}{hyp}}{\frac{adj}{hyp}}$

$= \frac{opp}{\cancel{hyp}} \times \frac{\cancel{hyp}}{adj}$

$= \frac{opp}{adj} = \tan\theta$!

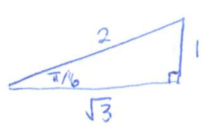
1. Given the identity $\sin\theta\cot\theta = \cos\theta$

a) Verify the identity for $\theta = \frac{\pi}{6}$.

$\sin\frac{\pi}{6} \cdot \cot\frac{\pi}{6} = \cos\frac{\pi}{6}$

$\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{1}\right) = \frac{\sqrt{3}}{2}$

$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$ ✓



b) Prove the identity.

LS	RS
$\cancel{\sin\theta}\left(\frac{\cos\theta}{\cancel{\sin\theta}}\right)$	$\cos\theta$
$\cos\theta$	$\cos\theta$

LS = RS

- choose the more complicated side to simplify.
- change $\cot\theta$ to $\frac{\cos\theta}{\sin\theta}$
- simplify
- re-write

c) What are the non-permissible values of θ?

$\cot\theta = \frac{\cos\theta}{\sin\theta} \Rightarrow \sin\theta \neq 0$

$\sin\theta = 0$ at $0, \pi, 2\pi, \ldots$ $\therefore \boxed{\theta \neq n\pi \ (n \in \mathbb{Z})}$

2. Prove the identity $\frac{\tan\theta}{\sec\theta} = \sin\theta$. What are the non-permissible values of θ?

LS	RS
$\frac{\tan\theta}{\sec\theta}$	$\sin\theta$
$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}}$	
$= \frac{\sin\theta}{\cancel{\cos\theta}} \cdot \frac{\cancel{\cos\theta}}{1}$	
$= \sin\theta$	

LS = RS

$\hookrightarrow \sec\theta = \frac{1}{\cos\theta} \therefore \cos\theta \neq 0$

$\cos\theta = 0$ at $\frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

$\boxed{\theta \neq \frac{\pi}{2} + n\pi \ (n \in \mathbb{Z})}$

- rewrite in terms of sinθ and cosθ
- multiply by the reciprocal
- simplify

3. Prove the identity $\cos\theta = \frac{1 + \cos\theta}{1 + \sec\theta}$.

LS	RS
$\cos\theta$	$\frac{1+\cos\theta}{1+\sec\theta}$
	$= \frac{(1+\cos\theta)}{(1+\frac{1}{\cos\theta})} \cdot \frac{\cos\theta}{\cos\theta}$
	$= \frac{(1+\cos\theta)\cos\theta}{\cos\theta + \frac{\cancel{\cos\theta}}{\cancel{\cos\theta}}}$
	$= \frac{\cancel{(1+\cos\theta)}\cos\theta}{\cancel{\cos\theta+1}}$
	$= \cos\theta$

LS = RS

- multiply by $\frac{\cos\theta}{\cos\theta} = 1$ to remove complex fraction
- simplify

4. Solve each equation for $0 \le x < 2\pi$.

a) $2\sin x = 3 + 2\csc x$

$\left(2\sin x = 3 + \frac{2}{\sin x}\right) \cdot \sin x$

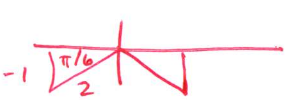
$2\sin^2 x = 3\sin x + 2$

$2\sin^2 x - 3\sin x - 2 = 0$

$(2\sin x + 1)(\sin x - 2) = 0$

$\sin x = -\frac{1}{2}$

$\sin x = 2$
no solution



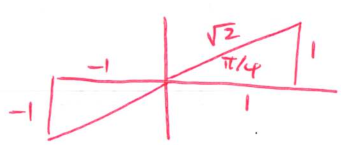
$\boxed{x = \frac{7\pi}{6}, \frac{11\pi}{6}}$

b) $\sin x = \cos x$

$\frac{\sin x}{\cos x} = \frac{\cos x}{\cos x}$

$\frac{\sin x}{\cos x} = 1$

$\tan x = 1$



$\boxed{x = \frac{\pi}{4}, \frac{5\pi}{4}}$

p. 6~~12~~53 # 3-6, 7ii, 8-11 and MC # 1, 2