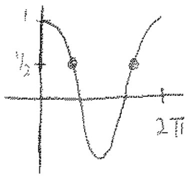
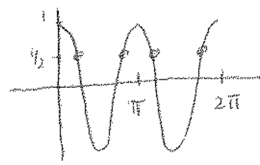
Pre-Calculus 12 Name: ____________________

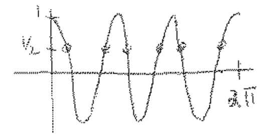
7.2 – Solving Trig Equations Algebraically (Part II) Block: __________

For $0 \le x < 2\pi$, how many solutions does each equation have?

a) $\cos x = \frac{1}{2}$ b) $\cos 2x = \frac{1}{2}$ c) $\cos 3x = \frac{1}{2}$ d) $\cos nx = \frac{1}{2}$







$\boxed{\text{2 solutions}}$ $\boxed{\text{4 solutions}}$ $\boxed{\text{6 solutions}}$ $\boxed{2n \text{ solutions}}$

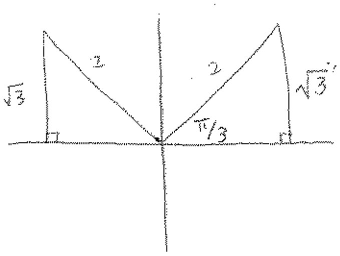
Solve each equation for $0 \le x < 2\pi$.

a) $\sin \boxed{2x} = \frac{\sqrt{3}}{2}$

period $= 2\pi\left(\frac{1}{2}\right) = \pi$

$2 \times 2 = 4$ solutions

S A
T C



$2x_1 = \frac{\pi}{3}$

$\Rightarrow \boxed{x_1 = \frac{\pi}{6}}$

$2x_2 = \pi - \frac{\pi}{3}$

$2x_2 = \frac{2\pi}{3}$

$\boxed{x_2 = \frac{2\pi}{6} = \frac{\pi}{3}}$

☆ Add the period (π) to find the next 2 solutions

$x_3 = \frac{\pi}{6} + \pi = \boxed{\frac{7\pi}{6}}$

$x_4 = \frac{\pi}{3} + \pi = \boxed{\frac{4\pi}{3}}$

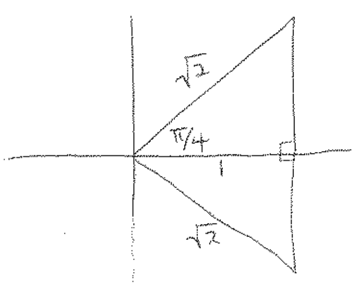
$\boxed{x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}}$

b) $\cos \boxed{3x} = \frac{1}{\sqrt{2}}$

period $= \frac{2\pi}{3}$

6 solutions

S A
T C



$3x_1 = \frac{\pi}{4}$

$\boxed{x_1 = \frac{\pi}{12}}$

$3x_2 = 2\pi - \frac{\pi}{4}$

$3x_2 = \frac{7\pi}{4}$

$\boxed{x_2 = \frac{7\pi}{12}}$

$x_3 = \frac{\pi}{12} + \frac{2\pi}{3} = \frac{\pi}{12} + \frac{8\pi}{12} = \frac{9\pi}{12} = \boxed{\frac{3\pi}{4}}$

$x_4 = \frac{7\pi}{12} + \frac{2\pi}{3} = \frac{7\pi}{12} + \frac{8\pi}{12} = \frac{15\pi}{12} = \boxed{\frac{5\pi}{4}}$

$x_5 = \frac{3\pi}{4} + \frac{2\pi}{3} = \frac{9\pi}{12} + \frac{8\pi}{12} = \boxed{\frac{17\pi}{12}}$

$x_6 = \frac{5\pi}{4} + \frac{2\pi}{3} = \frac{15\pi}{12} + \frac{8\pi}{12} = \boxed{\frac{23\pi}{12}}$

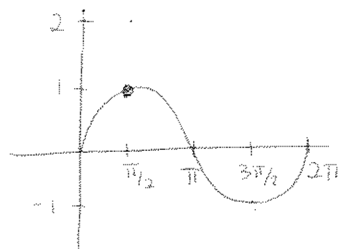
$\boxed{x = \frac{\pi}{12}, \frac{7\pi}{12}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{17\pi}{12}, \frac{23\pi}{12}}$

c) $\sin^2 x - 3\sin x + 2 = 0$ $\quad$ ★ Let $u = \sin x$

$\Rightarrow u^2 - 3u + 2 = 0$

$(u-2)(u-1) = 0$

$\Rightarrow u = 2,\ u = 1$



$\therefore \sin x = 2$, $\sin x = 1$

no solution! $\quad \boxed{x = \frac{\pi}{2}}$

$-1 \le \sin x \le 1$

d) $2\cos^2 x + \cos x - 1 = 0$ $\quad$ ★ Let $u = \cos x$

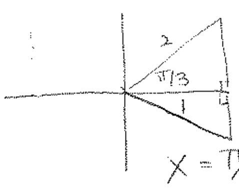
$\Rightarrow 2u^2 + u - 1 = 0$

$(2u-1)(u+1) = 0$

$\Rightarrow u = \frac{1}{2},\ u = -1$

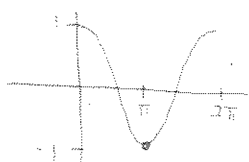
$\boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}, \pi}$

$\therefore \cos x = \frac{1}{2}$, $\cos x = -1$



$x = \pi/3$

$x = 5\pi/3$



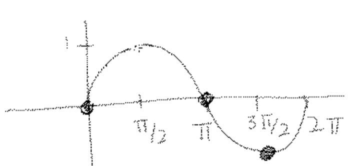
$\boxed{x = \pi}$

e) $\sin^2 x + \sin x = 0$

$\sin x(\sin x + 1) = 0$ $\quad$ or $\quad$ $u^2 + u = 0$

$u(u+1) = 0$

$\sin x = 0$, $\sin x = -1$ $\quad$ $u = 0,\ u = -1$



$\boxed{x = 0, \pi, \frac{3\pi}{2}}$

Worksheet # 13, 14, 19–24