Lesson 8.6 Exercises, pages 586–591

Exercises

A

3. Identify the transformations that would be applied to the graph of $y = \sin x$ to get the graph of $y = 10 \sin \frac{1}{3}(x - \pi) + 1$.

Compare $y = 10 \sin \frac{1}{3}(x - \pi) + 1$ with $y = a \sin b(x - c) + d$:

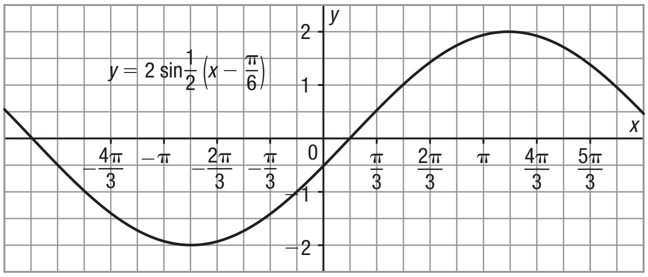
$a = 10$, so the graph of $y = \sin x$ is stretched vertically by a factor of 10.

$b = \frac{1}{3}$, so the graph of $y = \sin x$ is stretched horizontally by a factor of 3.

$c = \pi$, so the graph of $y = \sin x$ is translated π units right.

$d = 1$, so the graph of $y = \sin x$ is translated 1 unit up.

4. Identify the following characteristics of the graph below: amplitude, period, phase shift, equation of the centre line, zeros, domain, maximum value, minimum value, range



The amplitude is 2. The period is 4π. The phase shift is $\frac{\pi}{6}$.

The equation of the centre line is $y = 0$. The zeros are $-\frac{11\pi}{6}$ and $\frac{\pi}{6}$.

The graph is shown on domain $-2\pi \le x \le 2\pi$. The maximum value is 2.

The minimum value is -2. The range is $-2 \le y \le 2$.

B

5. Use the given data to write an equation for each function. Show your work.

a) a sine function with: amplitude 5, period 3π, equation of centre line $y = -2$, and phase shift $\frac{\pi}{3}$

Use: $y = a \sin b(x - c) + d$

Since the period $= \frac{2\pi}{b}$, then $b = \frac{2\pi}{3\pi}$, or $\frac{2}{3}$

In $y = a \sin b(x - c) + d$, substitute: $a = 5$, $b = \frac{2}{3}$, $c = \frac{\pi}{3}$, $d = -2$

An equation is: $y = 5 \sin \frac{2}{3}\left(x - \frac{\pi}{3}\right) - 2$

b) a cosine function with: maximum value 5, minimum value -2, period π, and phase shift $-\frac{\pi}{4}$

Use: $y = a \cos b(x - c) + d$

From the maximum and minimum values, $a = \frac{5 - (-2)}{2}$, or 3.5

From the period, $b = \frac{2\pi}{\pi}$, or 2

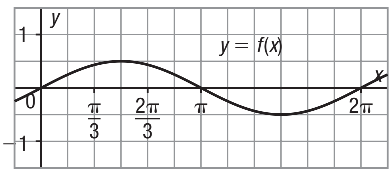
From the maximum value and the amplitude, $d = 5 - 3.5$, or 1.5

In $y = a \cos b(x - c) + d$, substitute: $a = 3.5$, $b = 2$, $c = -\frac{\pi}{4}$, $d = 1.5$

An equation is: $y = 3.5 \cos 2\left(x + \frac{\pi}{4}\right) + 1.5$

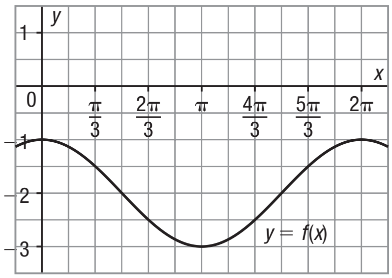
6. Determine a possible equation for each function graphed below.

a)

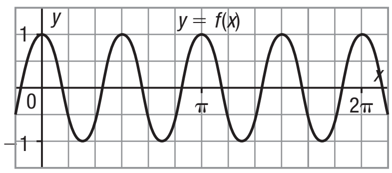


Sample response:
The graph is the image of $y = \sin x$ after a vertical compression by a factor of $\frac{1}{2}$.

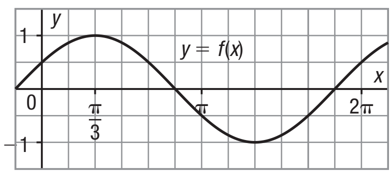
An equation is: $y = \frac{1}{2}\sin x$

b) 

Sample response:
The graph is the image of $y = \cos x$ after a vertical translation of 2 units down.
An equation is: $y = \cos x - 2$

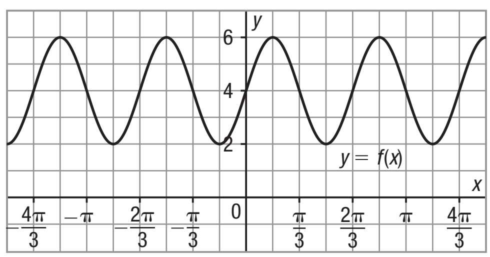
c) 

Sample response:
The graph is the image of $y = \cos x$ after a horizontal compression by a factor of $\frac{1}{4}$.
An equation is: $y = \cos 4x$

d) 

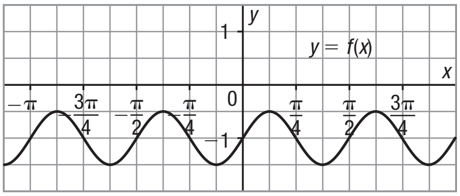
Sample response:
The graph is the image of $y = \cos x$ after a horizontal translation of $\frac{\pi}{3}$ units right.
An equation is: $y = \cos\left(x - \frac{\pi}{3}\right)$

7. a) For the function graphed below, identify the values of a, b, c, and d in $y = a \sin b(x - c) + d$, then write an equation for the function. Justify your answers.



Sample response:
The equation of the centre line is $y = 4$, so the vertical translation is 4 units up and $d = 4$.
The amplitude is: $\frac{6 - 2}{2} = 2$, so $a = 2$
Choose the x-coordinates of two adjacent maximum points, such as $\frac{\pi}{6}$ and $\frac{5\pi}{6}$. The period is: $\frac{5\pi}{6} - \frac{\pi}{6} = \frac{2\pi}{3}$
So, b is: $\frac{2\pi}{\frac{2\pi}{3}} = 3$
The sine function begins its cycle at $x = 0$; so the phase shift is 0, and $c = 0$.
Substitute for a, b, c, and d in: $y = a \sin b(x - c) + d$
An equation is: $y = 2 \sin 3x + 4$

b) For the function shown, identify the values of a, b, c, and d in $y = a \cos b(x - c) + d$, then write an equation for the function.



Sample response: The equation of the centre line is $y = -1$, so the vertical translation is 1 unit down and $d = -1$.

The amplitude is: $\frac{-0.5 - (-1.5)}{2} = 0.5$, so $a = \frac{1}{2}$

Choose the x-coordinates of two adjacent maximum points, such as $\frac{\pi}{8}$ and $\frac{5\pi}{8}$. The period is: $\frac{5\pi}{8} - \frac{\pi}{8} = \frac{\pi}{2}$

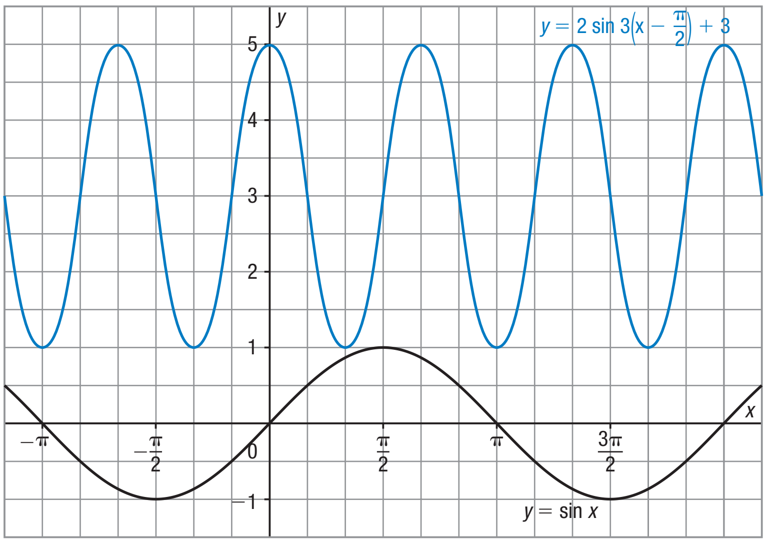
So, b is: $\frac{2\pi}{\frac{\pi}{2}} = 4$

To the right of the y-axis, the cosine function begins its cycle at $x = \frac{\pi}{8}$, so the phase shift is $\frac{\pi}{8}$, and $c = \frac{\pi}{8}$.

Substitute for a, b, c, and d in: $y = a \cos b(x - c) + d$

An equation is: $y = \frac{1}{2} \cos 4\left(x - \frac{\pi}{8}\right) - 1$

8. a) The graph of $y = \sin x$ is shown below. On the same grid, sketch the graph of $y = 2 \sin 3\left(x - \frac{\pi}{2}\right) + 3$. Describe your strategy.

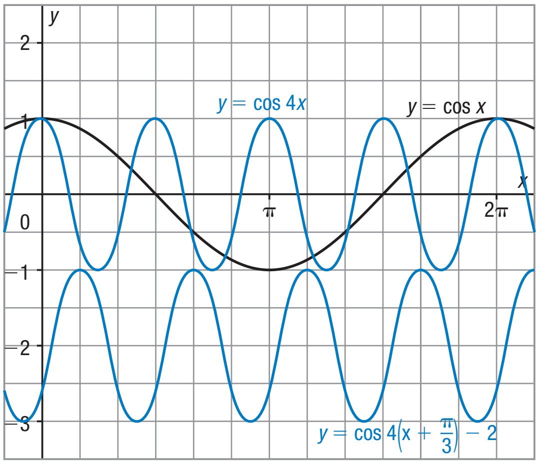


The graph of $y = \sin x$ is: stretched vertically by a factor of 2, compressed horizontally by a factor of $\frac{1}{3}$, then translated $\frac{\pi}{2}$ units right and 3 units up
I chose points on the graph of $y = \sin x$, applied the transformations to each point, then joined the image points.

b) List the characteristics of the function $y = 2 \sin 3\left(x - \frac{\pi}{2}\right) + 3$.

The amplitude is 2; the period is $\frac{2\pi}{3}$; the phase shift is $\frac{\pi}{2}$; the domain is $x \in \mathbb{R}$; the range is $1 \leq y \leq 5$; there are no zeros.

9. a) The graph of $y = \cos x$ is shown below. On the same grid, sketch the graph of $y = \cos 4\left(x + \frac{\pi}{3}\right) - 2$. Describe your strategy.



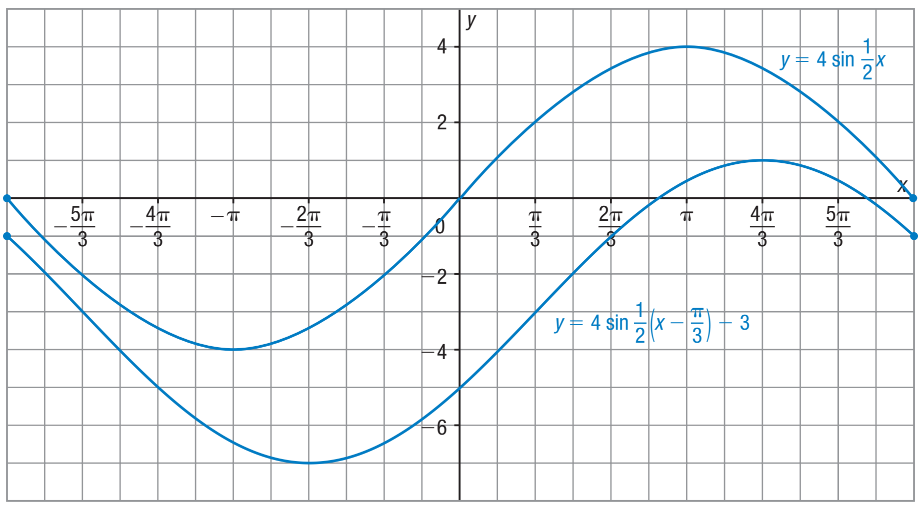
The graph of $y = \cos x$ is: compressed horizontally by a factor of $\frac{1}{4}$, then translated $\frac{\pi}{3}$ units left and 2 units down. I first graphed $y = \cos 4x$, then chose points on this graph and applied the remaining transformations to each point. I continued the pattern of image points, then joined them.

b) List the characteristics of the function $y = \cos 4\left(x + \frac{\pi}{3}\right) - 2$.

The amplitude is 1; the period is $\frac{2\pi}{4} = \frac{\pi}{2}$; the phase shift is $-\frac{\pi}{3}$; the domain is $x \in \mathbb{R}$; the range is $-3 \le y \le -1$; there are no zeros.

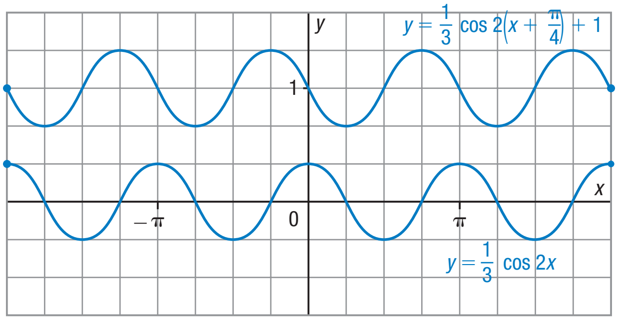
10. Sketch the graph of each function for the domain $-2\pi \le x \le 2\pi$.

a) $y = 4 \sin \frac{1}{2}\left(x - \frac{\pi}{3}\right) - 3$



Sketch the graph of $y = 4 \sin \frac{1}{2}x$, then translate it $\frac{\pi}{3}$ units right and 3 units down.

b) $y = \frac{1}{3} \cos 2\left(x + \frac{\pi}{4}\right) + 1$



Sketch the graph of $y = \frac{1}{3} \cos 2x$, then translate it $\frac{\pi}{4}$ units left and 1 unit up.

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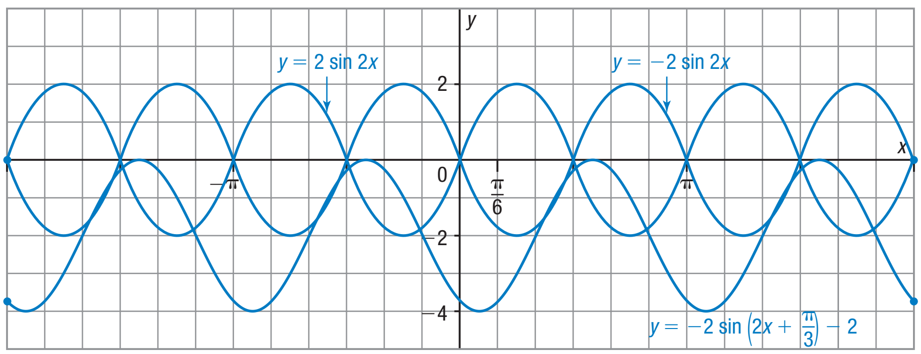
11. Use transformations to sketch the graph of $y = -2 \sin\left(2x + \frac{\pi}{3}\right) - 2$ for $-2\pi \leq x \leq 2\pi$.



Write the function as $y = -2 \sin 2\left(x + \frac{\pi}{6}\right) - 2$.

Sketch the graph of $y = 2 \sin 2x$, reflect it in the x-axis to get the graph of $y = -2 \sin 2x$, then translate this graph $\frac{\pi}{6}$ units left and 2 units down.