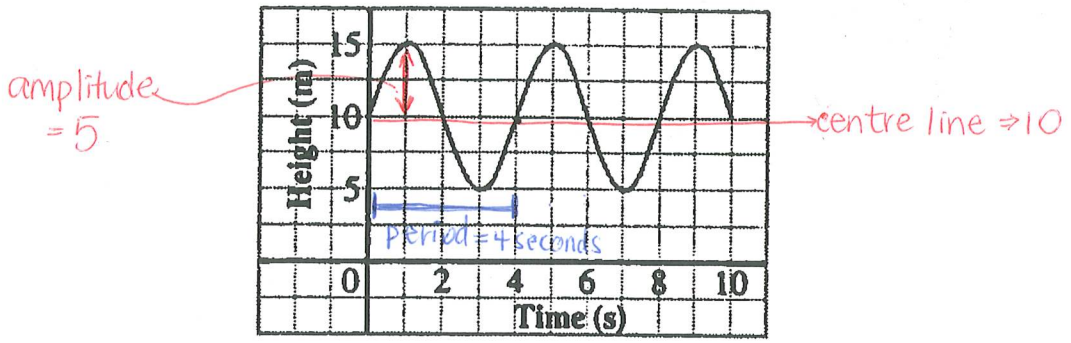
Pre-Calculus 12
8.5 – Trigonometric Functions

Name:_____
Block: _____

A function that repeats its values in regular intervals over its domain is called a **periodic function**.

The length of each interval, or *cycle*, measured along the horizontal axis is the **period** of the function.
e.g. The sine and cosine functions are periodic functions with period $2\pi$.

Functions whose graphs have the same shape as $y = \sin x$ or $y = \cos x$ are called **sinusoidal functions**.




Sinusoidal functions have maximum and minimum values that are equidistant from the centre line of the graph. The **amplitude** of the function is the distance of a max or min point from the centre line.

Ex. 1 Determine the amplitude of each function.

a) $y = 2\sin x$

↑ vertical expansion by 2

$y = \sin x$ has amplitude = 1

⇒ amplitude = 2

b) $y = \frac{2}{3}\cos x$

↑ vertical compression by $\frac{2}{3}$

max: $\frac{2}{3}$ min: $\frac{-2}{3}$

amplitude = $\frac{2}{3}$

c) $y = -4\sin x$

reflection in y-axis; ↑ vertical expansion by 4

amplitude = 4

*amplitude can never be negative!

**Note**: The tangent function is a periodic function with period $\pi$. It is not a sinusoidal function.
The graph of $y = \tan x$ has asymptotes at $= \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots = \frac{\pi}{2} + n\pi\ (n \epsilon Z)$.
The function has no maximum or minimum value, so the graph has no amplitude.

Ex. 2 Determine the period of each function.

a) $y = \cos 2x$

↑ hor. exp. by 2

period = $2\pi\left(\frac{1}{2}\right)$

= $\pi$

b) $y = \tan\frac{2}{3}x$

↑ hor. exp. by $\frac{3}{2}$

Period = $\pi\left(\frac{3}{2}\right)$

= $\frac{3\pi}{2}$

c) $y = \sin\frac{x}{7}$

↑ hor. exp. by 7

period = $2\pi(7)$

= $14\pi$

The period of $y = \sin bx$ and $y = \cos bx$ is $\frac{2\pi}{b}$ $(b > 0)$. The period of $y = \tan bx$ is $\frac{\pi}{b}$ $(b > 0)$.

When a graph has been translated horizontally, the distance of the translation is called the **phase shift** of the function.

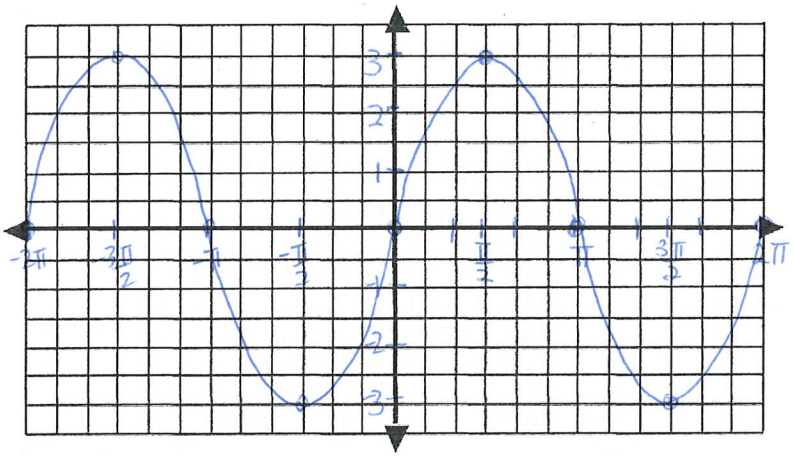
Ex. 3 Describe how the graph of each function relates to the original, and sketch the graph for $0 \le x \le 2\pi$.

a) $y = 3 \sin x$

amplitude = 3

(vertical expansion by 3)

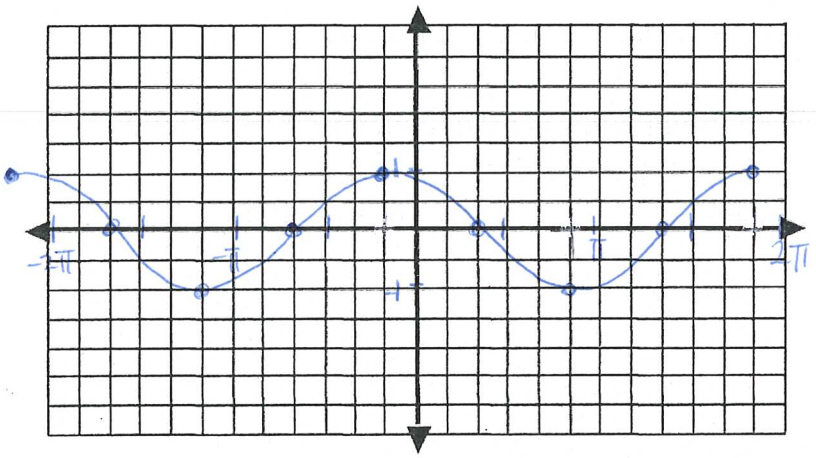
tip: plot the first and last points, then the middle. The last two points lie in the middle of those.



b) $y = \cos\left(x + \frac{\pi}{6}\right)$

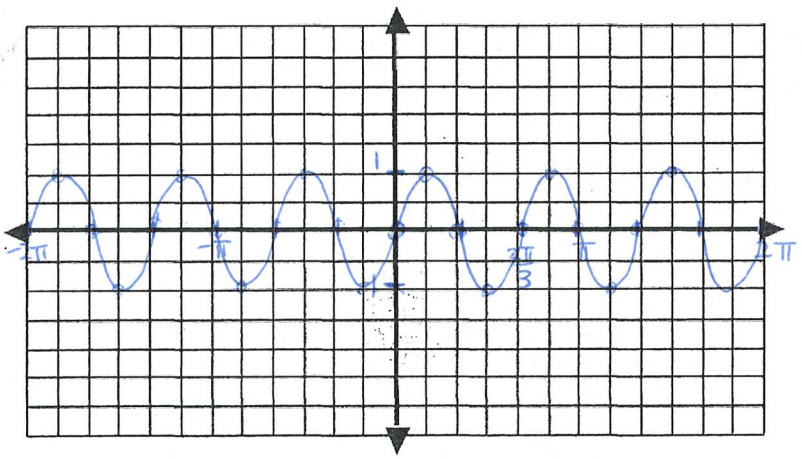
phase shift = $-\frac{\pi}{6}$

($\frac{\pi}{6}$ to the left)



c) $y = \sin 3x$ (b = 3)

period = $\frac{2\pi}{3}$



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