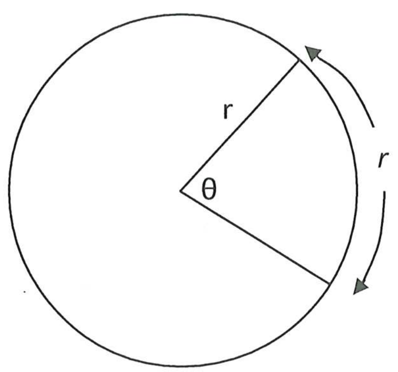
Pre-Calculus 12 Name:______________________________

6.3 (I) – Radian Measure Block: _____________

Degrees and radians are 2 different units used to measure angles, just as centimetres and inches are both used to measure length.

Definition: One **radian** is the measure of an angle subtended at the centre of a circle by an arc equal in length to the radius of the circle.

$\theta = 1$ radian



Note: $\frac{\theta}{r} = \frac{360°}{C} \Rightarrow \frac{\theta}{r} = \frac{360°}{2\pi r}$

Circumference

if $\theta = 1$ radian, then $\frac{1}{r} = \frac{360°}{2\pi r} \Rightarrow 1 \text{ radian} = \frac{360° r}{2\pi r}$

and so, $1 \text{ radian} = \frac{180°}{\pi} \sim 57°$

$$\boxed{1 \text{ radian} = \frac{180°}{\pi} \Rightarrow 1° = \frac{\pi}{180°} \text{ radians}}$$

Ex. 1. Convert to radians (leave in exact form).

a) $240\cancel{°}\left(\frac{\pi}{180\cancel{°}}\right) = \frac{240\pi}{180} = \boxed{\frac{4\pi}{3}}$

b) $360°\left(\frac{\pi}{180°}\right) = \frac{360\pi}{180} = \boxed{2\pi}$

$$\boxed{\begin{aligned} * \ 360° &= 2\pi \\ 180° &= \pi \end{aligned}}$$

Ex. 2. Convert to radians (leave in exact form).

a) $\frac{7\cancel{\pi}}{4}\left(\frac{180°}{\cancel{\pi}}\right) = 315°$

b) $-\frac{5\cancel{\pi}}{6}\left(\frac{180°}{\cancel{\pi}}\right) = -150°$

Recall: Arc length = radius when $\theta = 1$ radian.

So if $\theta > 1$, arc length > radius and if $\theta < 1$, then arc length < radius.

In general: $\boxed{\text{arc length } = r\theta \text{ (in radians)}}$.

Ex.4. Find the arc length of a circle with radius 5 cm subtended by a central angle of:

a) 0.8 radians

$a = 5(0.8)$

$= \boxed{4\text{cm}}$

b) $\frac{\pi}{3}$

$a = 5\left(\frac{\pi}{3}\right)$

$= \boxed{5.2\text{cm}}$

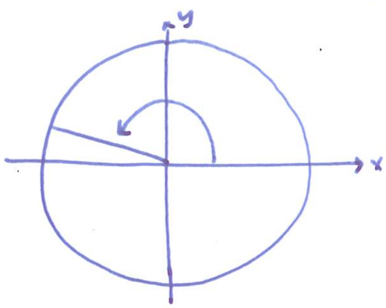
c) 100° convert first!

$100°\left(\frac{\pi}{180°}\right) = \frac{100\pi}{180} = \frac{5\pi}{9}$

$a = 5\left(\frac{5\pi}{9}\right) = \boxed{8.7\text{ cm}}$

Ex. 2. Sketch each angle in standard position, find 2 coterminal angles, and write an expression to represent all coterminal angles.

a) $\theta = 3$ rad. $3\left(\frac{180}{\pi}\right) \sim 172°$



add 2π to find coterminal angles!

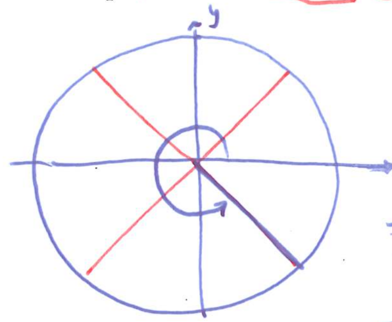
$3 + 2\pi = 9.28$

$3 - 2\pi = -3.28$

$\boxed{3 + 2\pi n \ (n \in \mathbb{Z})}$

b) $\theta = \frac{7\pi}{4}$ ← split into 4 equal pieces

Note: $\pi = 180°$

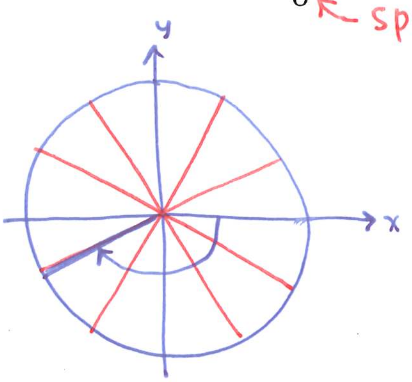


$\frac{7\pi}{4} + 2\pi = \frac{7\pi}{4} + \frac{8\pi}{4} = \frac{15\pi}{4}$

$\frac{7\pi}{4} - 2\pi = \frac{7\pi}{4} - \frac{8\pi}{4} = -\frac{\pi}{4}$

$\boxed{\frac{7\pi}{4} + 2\pi n \ (n \in \mathbb{Z})}$

c) $\theta = -\frac{5\pi}{6}$ ← split π into 6 pieces!

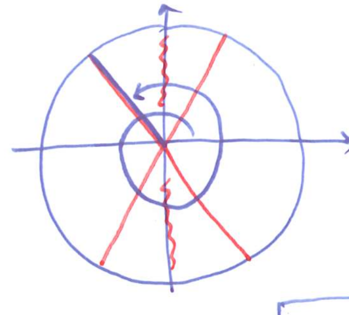


$-\frac{5\pi}{6} + 2\pi = -\frac{5\pi}{6} + \frac{12\pi}{6} = \frac{7\pi}{6}$

$-\frac{5\pi}{6} - 2\pi = -\frac{5\pi}{6} - \frac{12\pi}{6} = -\frac{17\pi}{6}$

$\boxed{-\frac{5\pi}{6} + 2\pi n \ (n \in \mathbb{Z})}$

d) $\theta = \frac{8\pi}{3}$



$\frac{8\pi}{3} + 2\pi = \frac{8\pi}{3} + \frac{6\pi}{3} = \frac{14\pi}{3}$

$\frac{8\pi}{3} - \frac{6\pi}{3} = \frac{2\pi}{3}$

$\boxed{\frac{8\pi}{3} + 2\pi n \ (n \in \mathbb{Z})}$

p. ~~494~~ 552 # 4 – 6, 8, 9a, 11, MC