5.8 Solving Problems with Exponents and Logarithms (Part II)
(This section is all about comparing values, rather than finding specific values.)

Recall: $P = P_0 (\text{rate})^{\text{time/period}}$

Population

Ex. 1) The population of a swarm of insects increases fivefold in 3 weeks. How many times greater is the population of the swarm after 6 weeks than it is after 4 weeks?

$$P = P_0 (5)^{t/3}$$

$$\frac{P = P_0 (5)^{6/3}}{P = P_0 (5)^{4/3}} = 5^{\frac{6}{3} - \frac{4}{3}} = 5^{\frac{6-4}{3}} = 5^{\frac{2}{3}} = \boxed{2.9 \text{ times}}$$

note: 6 weeks − 4 weeks = 2 weeks

Earthquakes

Each increase of 1 unit on the Richter scale represents a tenfold increase in intensity as measured on a seismometer.   rate = 10   period = 1

Ex. 2) The San Francisco earthquake of 1989 measured 6.9 on the Richter scale, while an earthquake on the B.C. coast in 1700 measured 9.0. How many times as intense as the San Francisco earthquake was the B.C. earthquake?

$$I = I_0 (10)^R$$

$$\Rightarrow \frac{I = I_0 (10)^{9.0}}{I = I_0 (10)^{6.9}} \Rightarrow 10^{9-6.9} = 10^{2.1} = \boxed{125.9 \text{ times}}$$

Ex. 3) Determine the magnitude of an earthquake that is $\frac{1}{4}$ times as intense as the B.C. earthquake of 1700?   less than 9

$$\frac{10^x}{10^{9.0}} = \frac{1}{4}$$

$$\log 10^{x-9} = \log \frac{1}{4}$$

$$(x-9)(\log 10) = \log \frac{1}{4}$$

$$\underset{= 1}{\smile}$$

$$x - 9 = \log \frac{1}{4}$$

$$x = \log \frac{1}{4} + 9$$

$$\boxed{x = 8.4}$$

Loudness

An increase of 10 decibels (dB) represents a tenfold increase in loudness.

rate = 10
period = 10

Ex. 4) Average street traffic measures 85 dB while a typical rock group measures 110 dB. How many times louder is the rock group than the street traffic?

$$L = L_0 (10)^{d/10}$$

$$\frac{L_0 (10)^{110/10}}{L_0 (10)^{85/10}} = 10^{\frac{110-85}{10}} = 10^{\frac{25}{10}} = \boxed{316.2 \text{ times}}$$

pH Levels

pH above 7:   each 1 unit increase in pH represents a tenfold increase in alkalinity

pH below 7:   each 1 unit decrease in pH represents a tenfold increase in acidity

rate = 10
period = 1

Ex. 5) Lemon juice has a pH level of 2, while tomato juice has a pH level of 4. How many times more acidic is lemon juice than tomato juice?

$$A = A_0 (10)^{pH}$$

$$\frac{A_0 (10)^4}{A_0 (10)^2} = 10^{4-2} = 10^2 = \boxed{100}$$

Ex. 6) Baking soda has a pH level of 9. Determine the pH level of a cleaner that is twice as alkaline as baking soda.

more than 9!

$$\frac{10^x}{10^9} = 2$$

$$\log 10^{x-9} = \log 2$$

$$(x-9) \log 10 = \log 2$$

$$= 1$$

$$x - 9 = \log 2$$

$$x = \log 2 + 9$$

$$\boxed{x = 9.3}$$