

Use exponent laws to simplify each expression:

a) $x^5 \cdot x^3 = x^8$

b) $\frac{x^8}{x^2} = x^6$

c) $(x^3)^4 = x^{12}$

1. Use exponent laws and the relationship between exponents and logarithms to evaluate each expression.

a) $\log_2 8 + \log_2 4 = 3 + 2 = 5$

note: $\log_2 (8 \times 4) = \log_2 32 = 5$

b) $\log_2 8 - \log_2 4 = 3 - 2 = 1$

note: $\log_2 (8 \div 4) = \log_2 2 = 1$

c) $2\log_2 8 = 2(3) = 6$

note: $\log_2 8^2 = \log_2 64 = 6$

Laws of Logarithms:

$\log_a x + \log_a y = \log_a xy$

$\log_a x - \log_a y = \log_a \frac{x}{y}$

$n\log_a x = \log_a x^n$

2. Evaluate each expression using the laws of logarithms.

a) $2\log_5 10 - 2\log_5 2$

$= \log_5 10^2 - \log_5 2^2$

$= \log_5 100 - \log_5 4$

$= \log_5 \frac{100}{4} \Rightarrow \log_5 25 = \boxed{2}$

$$\begin{aligned}
 \text{b) } \frac{1}{2} \log_3 45 - \frac{1}{2} \log_3 5 &= \log_3 45^{\frac{1}{2}} = \log_3 5^{\frac{1}{2}} \\
 &= \log_3 \sqrt{45} - \log_3 \sqrt{5} \\
 &= \log_3 \frac{\sqrt{45}}{\sqrt{5}} \Rightarrow \log_3 \sqrt{9} = \log_3 3 = \boxed{1}
 \end{aligned}$$

3. Use the laws of logarithms to write as a single logarithm.

$$\text{a) } 2 \log a + \frac{1}{4} \log b - 5 \log c = \log a^2 + \log b^{\frac{1}{4}} - \log c^5 = \boxed{\log \left(\frac{a^2 b^{\frac{1}{4}}}{c^5} \right)}$$

$$\begin{aligned}
 \text{b) } 5 \log a - 2 \log b + \frac{1}{3} \log c - \frac{1}{5} \log d &= \log a^5 - \log b^2 + \log c^{\frac{1}{3}} - \log d^{\frac{1}{5}} \\
 &= \boxed{\log \left(\frac{a^5 c^{\frac{1}{3}}}{b^2 d^{\frac{1}{5}}} \right)}
 \end{aligned}$$

4. Write in terms of $\log a$ and $\log b$.

$$\text{a) } \log \left(\frac{a^2}{b} \right) = \log a^2 - \log b = \boxed{2 \log a - \log b}$$

$$\begin{aligned}
 \text{b) } \log(100 \sqrt{a} b^3) &= \log 100 + \log \sqrt{a} + \log b^3 \quad \star \sqrt{a} = a^{\frac{1}{2}} \\
 &= \boxed{2 + \frac{1}{2} \log a + 3 \log b}
 \end{aligned}$$