Pre-Calculus 12 Name ____________________

6.2 – Geometric Series Block ______

A **geometric series** is the sum of the terms of a geometric sequence.

e.g. $2 + 6 + 18 + 54 + \ldots$.

This sum can be expressed symbolically as:

$$\boxed{S_n = \frac{t_1(r^n - 1)}{r-1} \quad (r \neq 1)} \quad \text{or} \quad \boxed{S_n = \frac{rt_n - t_1}{r-1} \quad (r \neq 1)}$$

The second formula is useful specifically when the last term t_n is known.

Example #1: Determine the sum of the first 10 terms of each geometric series. * use 1st formula

a) $5 + 15 + 45 + \ldots$

$t_1 = 5 \quad r = 3 \quad n = 10$

$$S_{10} = \frac{5(3^{10} - 1)}{3-1}$$

$$= \frac{5(59048)}{2}$$

$$= \boxed{147{,}620}$$

b) $t_1 = 64,\ r = \frac{1}{4}$ $\quad n = 10$

$$S_{10} = \frac{64\left(\frac{1}{4}^{10} - 1\right)}{\left(\frac{1}{4} - 1\right)}$$

$$= \boxed{85.33}$$

Example #2: Determine the sum of each geometric series. * use 2nd formula

a) $-2 + 4 - 8 + \ldots - 8192$

$t_1 = -2 \quad r = -2 \quad t_n = -8192$

$$S_n = \frac{[-2(-8192) - (-2)]}{[-2-1]}$$

$$= \frac{16384 + 2}{-3}$$

$$= \frac{16386}{-3}$$

$$= \boxed{-5462}$$

b) $\frac{1}{64} + \frac{1}{16} + \frac{1}{4} + \ldots + 1024$

$t_1 = \frac{1}{64} \quad r = 4 \quad t_n = 1024$

$$S_n = \frac{4(1024) - \frac{1}{64}}{4-1}$$

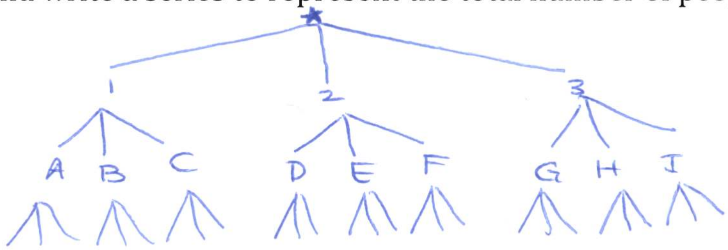
$$= \frac{\left(4096 - \frac{1}{64}\right)}{3}$$

$$= \boxed{1365.33}$$

Example #3: A phone tree is used to contact a large number of people in a short period of time. In a particular phone tree, the first person contacts 3 people, who each contact 3 more people, and so on.

a) Draw a diagram and write a series to represent the total number of people in the phone tree.

$1+3+9+27+\dots$



b) How many people are contacted after 6 levels of the tree (assuming the first level has 1 person)?

$t_1 = 1$

$r = 3$

$n = 6$

$$S_6 = \frac{1(3^6-1)}{3-1} = \frac{1(728)}{2} = \boxed{364 \text{ people}}$$

c) After how many levels will the total number of people contacted reach 2,391,484?

$t_1 = 1$

$r = 3$

$S_n = 2,391,484$

$$S_n = \frac{1(3^n-1)}{3-1} = 2,391,484$$

$$\frac{3^n-1}{2} = 2,391,484$$

$$3^n - 1 = 4,782,968$$

$3^n = 4,782,969$

Guess + check

$\boxed{n = 14 \text{ levels}}$

We can use **Sigma Notation** to represent a series. $\sum_{k=1}^{8} 5(4)^{k-1}$ means the sum from $k=1$ to $k=8$.

$\sum$ is the 18th capital letter in the Greek alphabet, corresponding to the letter S for word "sum".

Example #4: List each geometric series below, then determine the first term and common ratio.

a) $\sum_{k=1}^{5} 2(3)^{k-1} = 2+6+18+54+162$

$k=1 \Rightarrow 2(3)^{1-1} = 2(3)^0 = 2$

$k=2 \Rightarrow 2(3)^{2-1} = 2(3)^1 = 6$

$k=3 \Rightarrow 2(3)^{3-1} = 2(3)^2 = 18$

$k=4 \Rightarrow 2(3)^{4-1} = 2(3)^3 = 54$

$k=5 \Rightarrow 2(3)^{5-1} = 2(3)^4 = 162$

$\boxed{t_1 = 2 \quad r = 3}$

note:

$t_n = t_1 r^{n-1}$

$\boxed{t_n = 2(3)^{n-1}}$

b) $\sum_{k=1}^{8} 2^k = 2+4+8+16+32+64+128+256$

$k=1 \Rightarrow 2^1 = 2$

$k=2 \Rightarrow 2^2 = 4$

$k=3 \Rightarrow 2^3 = 8$

$k=4 \Rightarrow 2^4 = 16$

$k=5 \Rightarrow 2^5 = 32$

$k=6 \Rightarrow 2^6 = 64$

$k=7 \Rightarrow 2^7 = 128$

$k=8 \Rightarrow 2^8 = 256$

$\boxed{t_1 = 2 \quad r = 2}$

note: $t_n = 2(2)^{n-1}$

$= 2^1(2)^{n-1}$

$= 2^{1+n-1}$

$= 2^n$

Homework: p. 360 # 1 – 11, 13, 15, MC # 1, 2