Pre-Calculus 12 Name ______________________

6.1 – Geometric Sequences Block ______

Recall that in an **arithmetic sequence**, the difference of consecutive terms is constant.

Similarly, in a **geometric sequence**, the ratio of consecutive terms.

e.g. 2, 6, 18, 54, ...

Whereas arithmetic sequences contain a common difference between terms, geometric sequences contain a common ratio (r). This can be found by taking any term and dividing it by previous term.

The most general geometric sequence can be written as $t_1, t_1 r, t_1 r^2, t_1 r^3, \ldots$ with the general term represented by the equation:

$$\boxed{t_n = t_1 r^{n-1}}$$

Example #1: Suppose a colony of bearded ninja-Chickens doubles every year and that the colony starts at a population of 4.

a) Determine the general term to represent this situation.

$t_1 = 4$

$r = 2$

$\boxed{t_n = 4(2)^{n-1}}$

b) Determine the size of the colony after 12 years.

$t_{12} = 4(2)^{12-1}$

$= 4(2)^{11}$

$= \boxed{8192}$

c) After how many years will the population exceed 500 chickens?

$t_n = \frac{\cancel{4}(2)^{n-1}}{\cancel{4}} \geq \frac{500}{4}$

$2^{n-1} \geq 125$

$2^7 = 128$

$\Rightarrow \boxed{n = 8}$

$2^{8-1} = 2^7 = 128 \geq 125$

Example #2: Determine the missing value for each geometric sequence with the following properties.

a) If $n = 7$, $t_7 = 12288$, $t_1 = 3$, find r.

$t_n = 3r^{n-1} \Rightarrow t_7 = 3r^{7-1} = 12288$

$\frac{\cancel{3}r^6}{\cancel{3}} = \frac{12288}{3}$

$\sqrt[6]{r^6} = \sqrt[6]{4096}$

$\boxed{r = 4}$

b) If $n = 5$, $t_5 = \frac{128}{81}$, $r = \frac{4}{3}$, find t_1.

$t_n = t_1\left(\frac{4}{3}\right)^{n-1} \Rightarrow t_5 = t_1\left(\frac{4}{3}\right)^{5-1} = \frac{128}{81}$

$t_1\left(\frac{4}{3}\right)^4 = \frac{128}{81}$

$t_1\left(\frac{256}{81}\right) = \frac{128}{81}$

$\boxed{t_1 = \frac{1}{2}}$

Example #3: Determine the value of t_2, t_3, t_4, and t_5 for a geometric series with $t_1 = 5$ and $t_6 = -1215$

$t_n = 5r^{n-1}$

$\Rightarrow t_6 = 5r^{6-1} = -1215$

$\frac{\cancel{5}r^5}{\cancel{5}} = \frac{-1215}{5}$

$\sqrt[5]{r^5} = \sqrt[5]{-243}$

$\boxed{r = -3}$

$t_1 = 5$

$\boxed{\begin{array}{l} t_2 = -15 \\ t_3 = 45 \\ t_4 = -135 \\ t_5 = 405 \end{array}}$

$t_6 = -1215$

Example #4: In a geometric sequence, the third term is 54 and the sixth term is -1458. Determine the values of t_1 and r and list the first 4 terms of the sequence.

$t_3 \quad t_4 \quad t_5 \quad t_6$

$54 \cdot r \cdot r \cdot r = -1458$

$\frac{\cancel{54}r^3}{\cancel{54}} = \frac{-1458}{54}$

$\sqrt[3]{r^3} = \sqrt[3]{-27}$

$\boxed{r = -3}$

$t_3 = t_1(-3)^{3-1} = 54$

$t_1(-3)^2 = 54$

$\frac{t_1(\cancel{9})}{\cancel{9}} = \frac{54}{9}$

$\boxed{t_1 = 6}$

Example #5: A ball is dropped from a height of 4m. After each bounce, it rises to 60% of its previous height. $\Rightarrow 0.6$

This can be modeled by a geometric sequence: $4\cdot(0.6), 4\cdot(0.6)^2, 4\cdot(0.6)^3, \ldots$

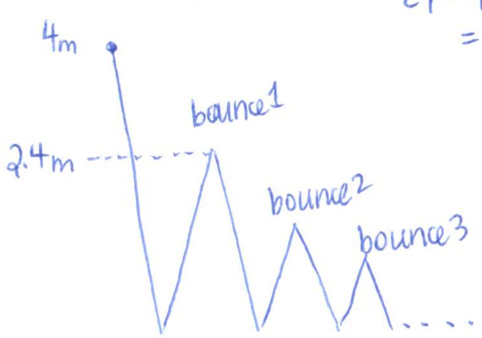
a) Write a general term for the sequence.

after bounce 1

$t_1 = 4 \times 0.6$

$= 2.4$

$r = 0.6$



$\boxed{t_n = 2.4(0.6)^{n-1}}$

b) What height does the ball reach after the 6th bounce?

$t_6 = 2.4(0.6)^{6-1}$

$= 2.4(0.6)^5$

$= \boxed{0.186624\text{m}}$

Homework: p. p.346 # 3-12, 14, 15, MC #1, 2