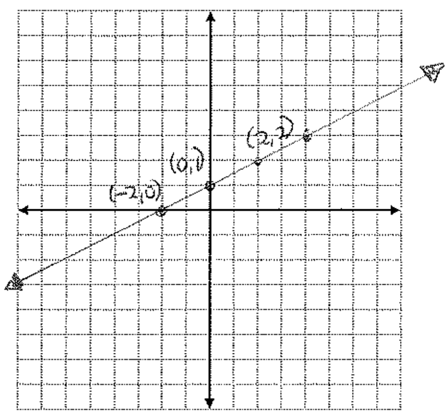
Pre–Calculus 12 Name ____________________

4.5 – Inverse Relations Block ______

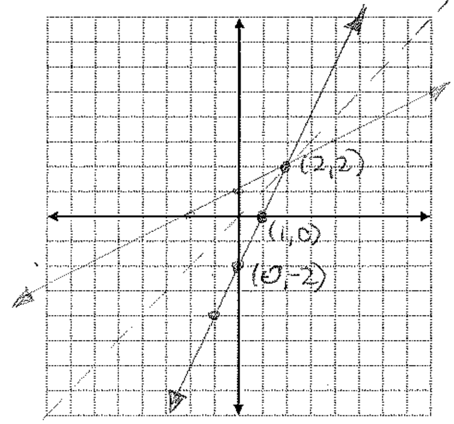
Consider the graph of $y = \frac{1}{2}x + 1$.

Label 3 different coordinates.



Reflect the graph in the line $y = x$.

Label 3 different coordinates.



Result (of coordinates): x and y coordinates switch! This is called the inverse.

The inverse equation is $x = \frac{1}{2}y + 1$. Solve the equation for y. Is the inverse a function? Yes

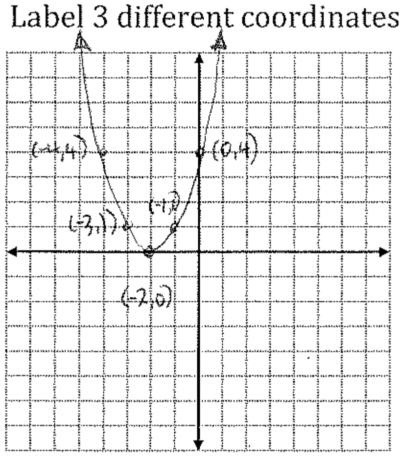
$\frac{x-1}{\frac{1}{2}} = \frac{\frac{1}{2}y}{\frac{1}{2}}$ $2(x-1) = y$

or $\boxed{y = 2x-2}$

We write this as $\boxed{f^{-1}(x) = 2x-2}$

Consider the graph of $y = (x+2)^2$

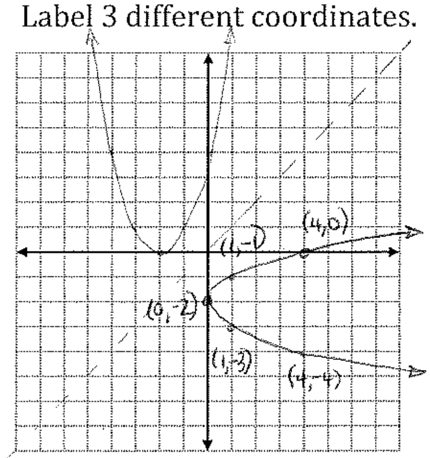
Label 3 different coordinates.



Domain: $x \in \mathbb{R}$ Range: $y \geq 0$

Reflect the graph in the line $y = x$.

Label 3 different coordinates.



Domain: $x \geq 0$ Range: $y \in \mathbb{R}$

Result (of coordinates): x and y switch Result of Domain and Range: they switch!

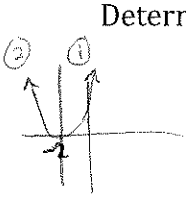
The inverse equation is $x = (y+2)^2$. Solve the equation for y. Is the inverse a function? NO

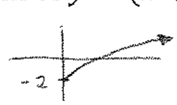
Doesn't pass the vertical line test.

$\sqrt{x} = \sqrt{(y+2)^2}$

$\pm\sqrt{x} = y+2$ $\boxed{y = \pm\sqrt{x} - 2}$

Determine 2 ways to restrict the domain of $y = (x+2)^2$ to make the inverse a function.



① $x \geq -2$ $\Rightarrow$ 

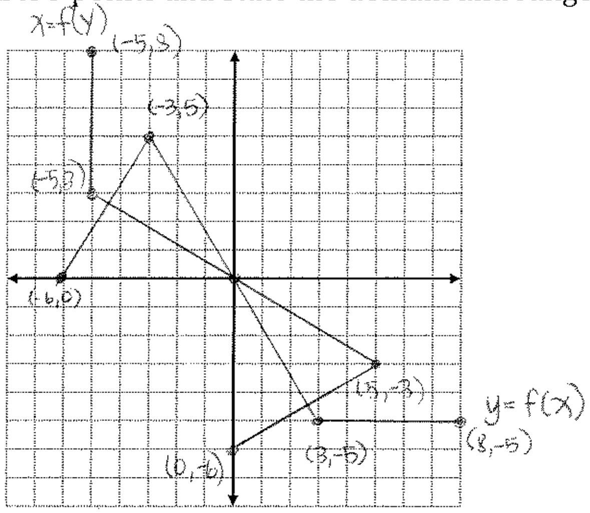
$\Rightarrow$ $f^{-1}(x) = \sqrt{x} - 2$

② $x \leq -2$ $\Rightarrow$ 

$\Rightarrow$ $f^{-1}(x) = -\sqrt{x} - 2$

- For a funtion $y = f(x)$, the graph $x = f(y)$ is the image of the graph after **reflection in the line $y = x$.**
- $y = f(x)$ and $x = f(y)$ are **inverses** of each other.
- A point (x, y) on $y = f(x)$ corresponds to the point (y, x) on $x = f(y)$.
- The domain of $y = f(x)$ is the range of $x = f(y)$ and the range of $y = f(x)$ is the domain of $x = f(y)$.
- If $x = f(y)$ is a function, $f^{-1}(x)$ is used to denote the inverse function.

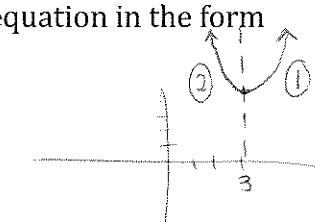
Example #1: Sketch a function, $y = g(x)$, using 3 straight lines. On the same grid, also graph the inverse. Label 3 points and state the domain and range of each. Is the inverse a function?



	$y = g(x)$	$x = g(y)$
Domain:	$-6 \le x \le 8$	$-5 \le x \le 5$
Range:	$-5 \le y \le 5$	$-6 \le y \le 8$

Is the inverse a function? NO

Example #2: Determine the equation of the inverse of each function. State the equation in the form $f^{-1}(x)$, restricting the domain of the original function if needed.



a) $y = \frac{3x-5}{2}$

$x = \frac{3y-5}{2}$

$2x = 3y - 5$

$\frac{2x+5}{3} = \frac{3y}{3} \Rightarrow y = \frac{2x+5}{3}$

$f^{-1}(x)$= $\frac{2x+5}{3}$

b) $y = 2(x-3)^2 + 4$

$x = 2(y-3)^2 + 4$

$\frac{x-4}{2} = \frac{2(y-3)^2}{2}$

$\sqrt{\frac{x-4}{2}} = \sqrt{(y-3)^2}$

$\pm\sqrt{\frac{x-4}{2}} = y - 3 \Rightarrow y = \pm\sqrt{\frac{x-4}{2}} + 3$

$f^{-1}(x)$= $\sqrt{\frac{x-4}{2}} + 3$ $\quad x \ge 3$

$f^{-1}(x)$= $-\sqrt{\frac{x-4}{2}} + 3$ $\quad x \le 3$

Homework: p. ~~243~~ 259 # 4-12