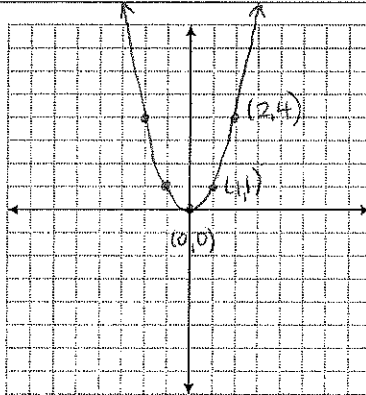


4.3 - Expanding & Compressing Graphs of Functions

Today we looked at **expanding** and **compressing** graphs of functions.

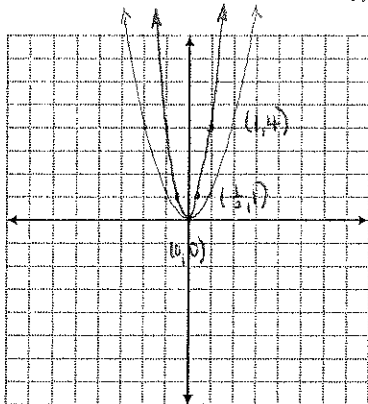
This type of transformation is a kind of horizontal or vertical "stretch" or "squish".

Consider the graph of $y = x^2$.



a) Replace "x" with "2x" and re-graph.

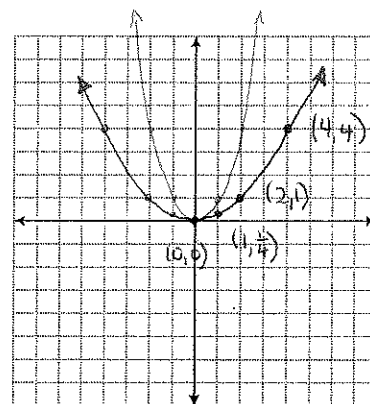
$$\Rightarrow y = (2x)^2$$



x-values are halved

b) Replace "x" with " $\frac{1}{2}x$ " and re-graph.

$$\Rightarrow y = \left(\frac{1}{2}x\right)^2$$



x-values are doubled

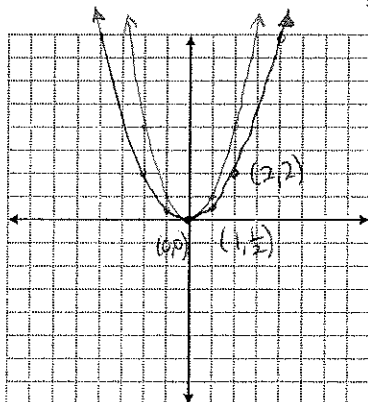
Result: horizontal compression by $\frac{1}{2}$

Result: horizontal expansion by 2

c) Replace "y" with "2y" and re-graph.

$$\Rightarrow 2y = x^2$$

$$y = \frac{1}{2}x^2$$

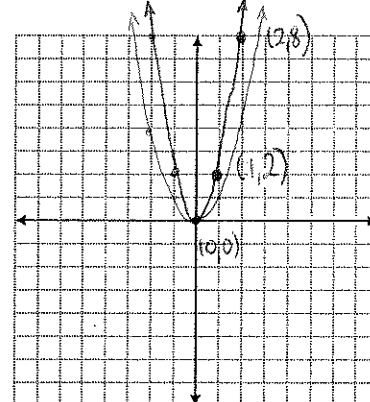


y-values are halved

d) Replace "y" with " $\frac{1}{2}y$ " and re-graph.

$$\Rightarrow \frac{1}{2}y = x^2$$

$$y = 2x^2$$



y-values are doubled

Result: vertical compression by $\frac{1}{2}$

Result: vertical expansion by 2

For the graph $y = af(x)$:

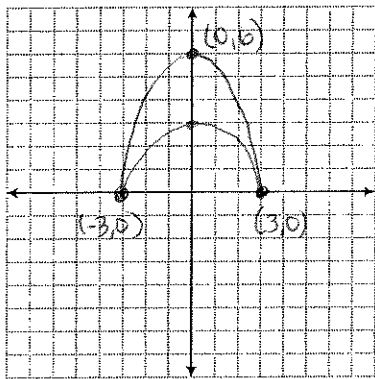
- When $0 < |a| < 1$, there is a vertical compression by a factor of $|a|$
- When $|a| > 1$, there is a vertical expansion (stretch) by a factor of $|a|$

For the graph $y = f(bx)$:

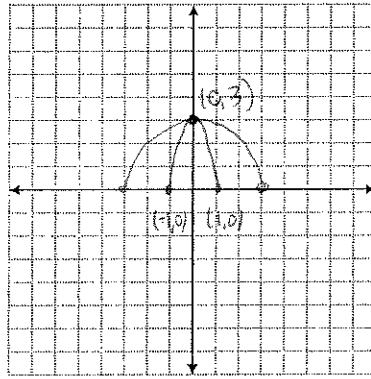
- When $0 < |b| < 1$, there is a horizontal expansion (stretch) by a factor of $\frac{1}{|b|}$
- When $|b| > 1$, there is a horizontal compression by a factor of $\frac{1}{|b|}$

Example #1: Draw $y = g(x)$, a semi-circle with radius 3 centered at the origin. On the same grid, also graph:

a) $y = 2g(x)$ *y-values doubled*



b) $y = g(3x)$ *x-values are tripled*



Result: vertical expansion by 2

Result: horizontal compression by $\frac{1}{3}$

Example #2: If the point $(4, -3)$ is on the graph of $y = g(x)$, find the corresponding point on the graph of:

a) $y = \frac{1}{3}g(x)$ *vertical compression by $\frac{1}{3}$*
 $(4, -1)$

b) $y = 3g\left(\frac{1}{2}x\right)$ *vertical expansion by 3 and horizontal expansion by 2*
 $(8, -9)$

Example #3: Sketch and label the graphs of each set of functions on the same grid.

$y = \sqrt{x}$

HE by 2
 $y = \sqrt{\frac{1}{2}x}$

$3y = \sqrt{x}$

VC by $\frac{1}{3}$
 $\Rightarrow y = \frac{1}{3}\sqrt{x}$

