

3.8 Factoring Special Polynomials

Ex.1) Expand and simplify:

a) $(x+2)(x-2)$
 $= x^2 - \cancel{2x} + \cancel{2x} - 4$
 $= \boxed{x^2 - 4}$

b) $(2x-3)(2x+3)$
 $= 4x^2 + \cancel{6x} - \cancel{6x} - 9$
 $= \boxed{4x^2 - 9}$

One binomial is the conjugate of another binomial if it has the same two terms, but the opposite sign in between them. When we multiply two conjugates, the result is a difference of squares. To factor a difference of squares, we use the pattern below:

$$\boxed{a^2 - b^2 = (a+b)(a-b)}$$

Ex.2) Factor:

a) $x^2 - 16$
 $\begin{array}{r} x \cdot x \\ - 4 \cdot 4 \\ \hline a) x^2 - 16 \end{array}$
 $= \boxed{(x+4)(x-4)}$

b) $4x^2 - 25$
 $\begin{array}{r} 2x \cdot 2x \\ - 5 \cdot 5 \\ \hline b) 4x^2 - 25 \end{array}$
 $= \boxed{(2x+5)(2x-5)}$

c) $81x^2 - 100y^2$
 $\begin{array}{r} 9x \cdot 9x \\ - 10y \cdot 10y \\ \hline c) 81x^2 - 100y^2 \end{array}$
 $= \boxed{(9x+10y)(9x-10y)}$

d) $4x^2 + 25$
impossible!

e) $x^4 - 16$
 $\begin{array}{r} x \cdot x \\ - 4 \cdot 4 \\ \hline e) x^4 - 16 \end{array}$
 $= (x^2 + 4)(x^2 - 4) \leftarrow \text{can factor!}$
 $= \boxed{(x^2 + 4)(x+2)(x-2)}$

f) $162a^4 - 2b^4$ Factor out a GCF of 2 first!
 $\begin{array}{r} 9a^2 \cdot 9a^2 \\ - b^2 \cdot b^2 \\ \hline f) 162a^4 - 2b^4 \end{array}$
 $= 2(81a^4 - b^4)$
 $= 2(9a^2 + b^2)(9a^2 - b^2) \leftarrow \text{can factor!}$
 $= \boxed{2(9a^2 + b^2)(3a+b)(3a-b)}$

Ex.3) Expand and simplify:

a) $(x+3)^2$
 $= (x+3)(x+3)$

$$= x^2 + 3x + 3x + 9$$

$$= \boxed{x^2 + 6x + 9}$$

b) $(2x+5)^2$
 $= (2x+5)(2x+5)$
 $= 4x^2 + 10x + 10x + 25$
 $= \boxed{4x^2 + 20x + 25}$

c) $(x-5)^2 = (x-5)(x-5)$
 $= x^2 - 5x - 5x + 25$
 $= \boxed{x^2 - 10x + 25}$

d) $(3x-4y)^2$
 $= (3x-4y)(3x-4y)$
 $= 9x^2 - 12xy - 12xy + 16y^2$
 $= \boxed{9x^2 - 24xy + 16y^2}$

The result of squaring a binomial is called a perfect square trinomial. A trinomial square can be factored according to the following patterns:

$$\boxed{a^2 + 2ab + b^2 = (a+b)(a+b) = (a+b)^2}$$

$$\boxed{a^2 - 2ab + b^2 = (a-b)(a-b) = (a-b)^2}$$

Ex.4) Identify and factor the perfect square trinomials:

a) $x \cdot x \quad 2 \cdot 4 \quad 4 \cdot 4$
 $x^2 + 8x + 16$
 $= (x+4)(x+4)$
 $= \boxed{(x+4)^2}$

b) $x \cdot x \quad 2x \cdot 5 \quad 5 \cdot 5$
 $x^2 - 10x + 25$
 $= (x-5)(x-5)$
 $= \boxed{(x-5)^2}$

c) $x \cdot x \quad 2x \cdot 6 \quad 6 \cdot 6$
 $x^2 + 12x + 36$
 \downarrow
 $12x$

d) $2x \cdot 2x \quad 5 \cdot 5$
 $4x^2 + 20x + 25$
 $\quad \quad 2 \cdot 2x \cdot 5$
 $= (2x+5)(2x+5)$
 $= \boxed{(2x+5)^2}$

not a perfect square!

e) $6x \cdot 6x \quad 1 \cdot 1$
 $36x^2 + 12x + 1$
 $\quad \quad 2 \cdot 6x \cdot 1$
 $= (6x+1)(6x+1)$
 $= \boxed{(6x+1)^2}$

f) $4x \cdot 4x \quad 7y \cdot 7y$
 $16x^2 + 56xy + 49y^2$
 $\quad \quad 2 \cdot 4x \cdot 7y$
 $= (4x+7y)(4x+7y)$
 $= \boxed{(4x+7y)^2}$