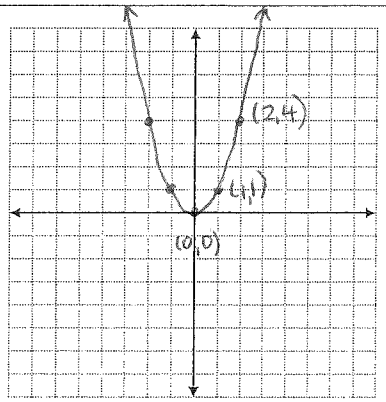
Pre-Calculus 12 Name _____

3.3 – Expanding & Compressing Graphs of Functions Block _____

Today we looked at **expanding** and **compressing** graphs of functions.

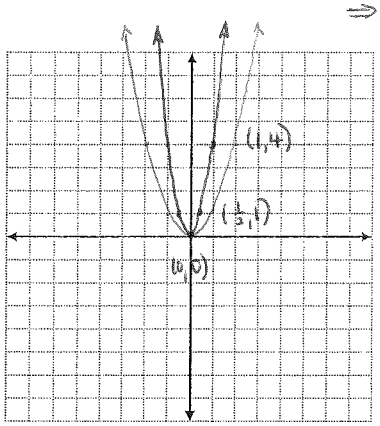
This type of transformation is a kind of horizontal or vertical "stretch" or "squish".

Consider the graph of $y = x^2$.



a) Replace "x" with "$2x$" and re-graph.

$\Rightarrow y = (2x)^2$

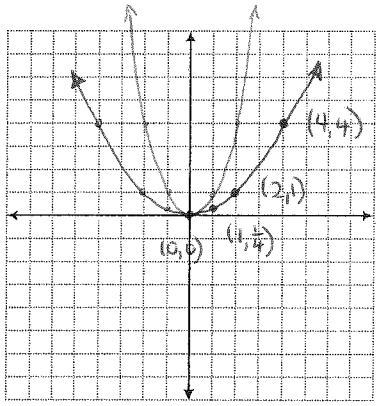


x-values are halved

Result: horizontal compression by $\frac{1}{2}$

b) Replace "x" with "$\frac{1}{2}x$" and re-graph.

$\Rightarrow y = \left(\frac{1}{2}x\right)^2$



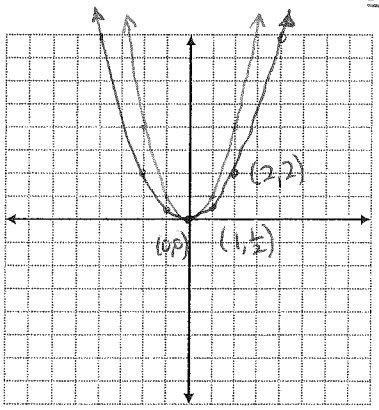
x-values are doubled

Result: horizontal expansion by 2

c) Replace "y" with "$2y$" and re-graph.

$\Rightarrow 2y = x^2$

$y = \frac{1}{2}x^2$



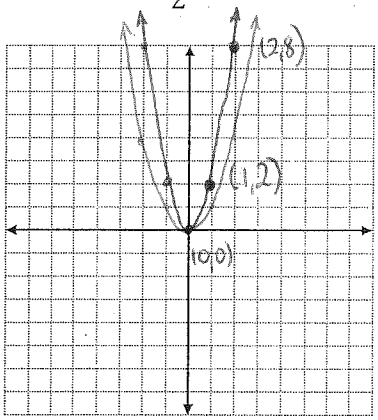
y-values are halved

Result: vertical compression by $\frac{1}{2}$

d) Replace "y" with "$\frac{1}{2}y$" and re-graph.

$\Rightarrow \frac{1}{2}y = x^2$

$y = 2x^2$



y-values are doubled

Result: vertical expansion by 2

For the graph $y = af(x)$:

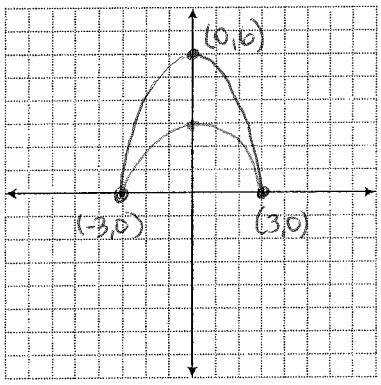
- When $0 < |a| < 1$, there is a vertical compression by a factor of $|a|$
- When $|a| > 1$, there is a vertical expansion (stretch) by a factor of $|a|$

For the graph $y = f(bx)$:

- When $0 < |b| < 1$, there is a horizontal expansion (stretch) by a factor of $\frac{1}{|b|}$
- When $|b| > 1$, there is a horizontal compression by a factor of $\frac{1}{|b|}$

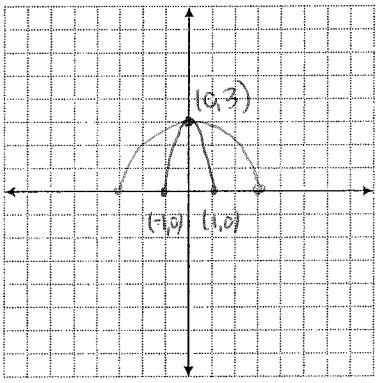
Example #1: Draw $y = g(x)$, a semi-circle with radius 3 centered at the origin. On the same grid, also graph:

a) $y = 2g(x)$ y-values doubled



Result: vertical expansion by 2.

b) $y = g(2x)$ x-values



Result: horizontal compression by $\frac{1}{2}$.

Example #2: If the point $(4, -3)$ is on the graph of $y = g(x)$, find the corresponding point on the graph of:

a) $y = \frac{1}{3}g(x)$ vertical compression by $\frac{1}{3}$

$\boxed{(4, -1)}$

b) $y = 3g\left(\frac{1}{2}x\right)$ vertical expansion by 3 and horizontal expansion by 2.

$\boxed{(8, -9)}$

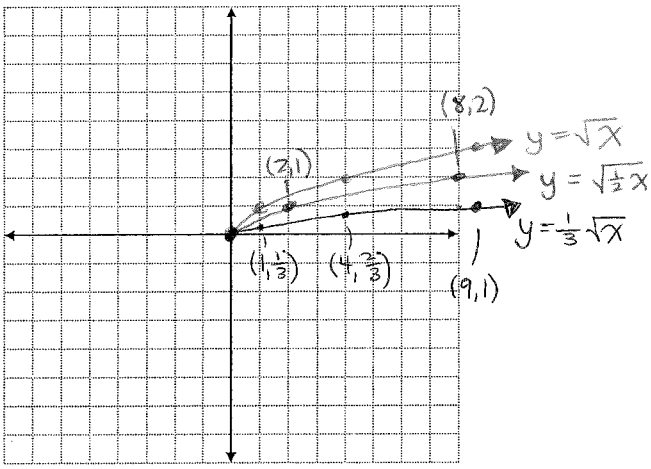
Example #3: Sketch and label the graphs of each set of functions on the same grid.

$y = \sqrt{x}$

HE by 2 $y = \sqrt{\frac{1}{2}x}$

$3y = \sqrt{x}$

VC by $\frac{1}{3}$ $\Rightarrow y = \frac{1}{3}\sqrt{x}$



Homework: p. 201 #2-14 (not 8, 11)